Global Analysis and Discrete Mathematics
Volume 6, Issue 1, pp. 109–129
ISSN: 2476-5341

Soft G-Metric Spaces for Fixed Point Theorems

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Received: 12 March 2021 / Accepted: 23 June 2021

Abstract In this article, we the concept of soft G-metric space and continuous soft mapping on it introduce. We also the Banach fixed point theorem in complete soft G-metric spaces investigate.

Keywords Soft G-metric space · Fixed point · Banach fixed point theorem · Soft set · G-Cauchy soft sequence · G-convergent soft sequence

Mathematics Subject Classification (2010) 47H10 · 54H25

1 Introduction

In [2], the concept of soft sets for dealing with uncertain objects as a general mathematical tool introduced. In [1], several basic notions of soft set theory were defined and studied. Later, in [4] the concepts of soft closed sets, soft open sets, soft closure, soft separation and soft interior axioms were introduced.

In this paper, we bring basic definitions of soft sets and we define soft G-metric on soft sets and we soft and complete soft G-metric spaces introduce.

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Also, we consider continuous soft mappings on it. First, we have the following definitions from [8]:

Definition 1 [8]. Let (F, A) and (F', A') are soft sets on X and Y, respectively. Let $f_1 : A \longrightarrow A'$ and $f_2 : F(A) \longrightarrow F'(A')$ be mappings, where $F(A) = \{F(a) : a \in A\}$ and $F'(A') = \{F'(a') : a' \in A'\}$. Then we define $f = (f_1, f_2) : (F, A) \longrightarrow (F', A')$ by

$$f(V,B)(e') = \begin{cases} \bigcup_{e \in f_1^{-1}(e') \cap B} f_2(V(e)) & f_1^{-1}(e') \cap B \neq \emptyset \\ \emptyset & \textit{otherwise.} \end{cases}$$

So it is said to be f a soft mapping and $(f(V, A), C)$ is a soft image of a soft set (V, B). Let $(W, C) \in (F', A')$, in which case the pre-image of (W, C) under the soft mapping f defined above is the soft set on X, denoted by $f^{-1}(W, C)$, where

$$f^{-1}(W,C)(e) = \begin{cases} f_2^{-1}(W(f_1(e))) & f_1(e) \in C \\ \emptyset & \textit{otherwise.} \end{cases}$$

Definition 2 [8]. Suppose $A \subseteq E$ a set of parameters, (a, r) and (b, r') be the parametric soft scalars. Therefore, the sum between soft parametric scalars and scalar multiplication on soft parametric scalars define as follows

$$(a, r) \dot{+} (b, r') = (\{a, b\}, r + r'), \text{ and } \lambda(a, r) = (a, \lambda r),$$

for every $\lambda \in \mathbb{R}$.

Definition 3 [8]. Let (F, A) be a soft set on X. We say a function f on (F, A) is parametric scalar valued, if there is functions $f_1 : A \longrightarrow A$ and $f_2 : F(A) \longrightarrow \mathbb{R}$ such that $f(F, A) = (f_1, f_2)(A, F(A))$.

Similarly, in [8] the parametric scalar valued function defined above is extended as $f : (F, A) \times (F, A) \longrightarrow (A, \mathbb{R})$ by $f(A \times A, F(A) \times F(A)) = (f_1, f_2)(A \times A, F(A) \times F(A))$, where $f_1 : A \times A \longrightarrow A$ and $f_2 : F(A) \times F(A) \longrightarrow \mathbb{R}$.

Definition 4 [3]. Consider nonempty set X. Let $G : X^3 \to [0, +\infty)$, therefore, G is called a metric on X if it has the following conditions:

(G1) $G(x^1, x^2, x^3) = 0$ if $x^1 = x^2 = x^3$.

(G2) $0 < G(x^1, x^1, x^2)$ for all $x^1, x^2 \in X$ with $x^1 \neq x^2$.

(G3) $G(x^1, x^1, x^2) \leq G(x^1, x^2, x^3)$ for all $x^1, x^2, x^3 \in X$ with $x^3 \neq x^2$.

(G4) $G(x^1, x^2, x^3) = G(x^1, x^3, x^2) = G(x^2, x^3, x^1) = \dots$ (symmetry in all three variables).

(G5) $G(x^1, x^2, x^3) \leq G(x^1, a, a) + G(a, x^2, x^3)$ for all $x^1, x^2, x^3, a \in X$, (rectangle inequality).

Then, we say (X, G) is a G-metric space.

Now, we give the definition of a soft G-metric space.

Definition 5 Let (F, A) be a soft set on X. Let $\widetilde{\pi} : A \times A \longrightarrow A$ be a parametric function. We call the parametric scalar valued function $\widetilde{G} : (F, A) \times (F, A) \times (F, A) \longrightarrow (A, \mathbb{R}^{+} \cup \{0\})$ is a soft metric on (F, A) if $\widetilde{G}$ satisfies in the following properties:

$$1. \widetilde{G}((a^1, F(a^1)), (a^2, F(a^2)), (a^3, F(a^3))) \succeq (\widetilde{\pi}(a^1, a^2, a^3), \widetilde{\pi}(a^1, a^2, a^3), 0),$$

and equality holds, whenever $a^1 = a^2 = a^3$.

$$2. \widetilde{G}((a^1, F(a^1)), (a^2, F(a^2)),(a^3, F(a^3))) = \widetilde{G}((a^1, F(a^1)), (a^3, F(a^3)), (a^2, F(a^2))) = \dots,$$

for all $a^1, a^2, a^3 \in A$.

$$3. \widetilde{G}((a^1, F(a^1)), (a^2, F(a^2))(a^3,F(a^3))) \preceq \widetilde{G}((a^1, F(a^1)), (d, F(d))(d, F(d))) \dot{+} \widetilde{G}((d, F(d)), (a^2, F(a^2)), (a^3, F(a^3))),$$

for all $a^1, a^2, a^3, d \in A$. We say the pair $((F, A), \widetilde{G})$ is a soft G-metric space on X.

Definition 6 Consider the soft set (F, A) on X. We say every element $(a^3, F(a^3)) \in (F, A)$ a soft point of (F, A) where $a^3 \in A$. in general terms if for some $a^3 \in A$, we have $x^1 \in F(a^3)$It will not necessarily be (a^3, x^1) in (F, A) but if it is in (F, A), then we call (a^3, x^1) a soft point (F, A)

Definition 7 Consider the soft set (F, A) on X, and $\widetilde{G}$ be a soft G-metric on (F, A). For each soft point $(a^3, x^1) \in (F, A)$, we show a distance of (a^3, x^1) from $(a^1, F(a^1))$ by

$$d_{\widetilde{G}}((a^3, x^1), (a^1, F(a^1))) = \widetilde{G}((a^3, x^1), (a^1, F(a^1)), (a^1, F(a^1))) \dot{+} \widetilde{G}((a^1, F(a^1)), (a^3, x^1), (a^3, x^1)).$$

If X be a metric space with metric d, then the distance (a^3, x^1) from $(a^1, F(a^1))$ can be defined as follows

$$\widetilde{G}((a^3, x^1), (a^1, F(a^1)),(a^1, F(a^1))) = (\widetilde{\pi}(a^3, a^1, a^1), \widetilde{\pi}(a^3, a^1, a^1), \inf_{x^2 \in F(a^1)} d(x^1, x^2)).$$

Definition 8 Consider the soft set (F, A) on X, let $\widetilde{G}$ be a soft G-metric on (F, A) and $r > 0$ be a real number. The soft ball with radius (a'^1, r) around $(a^1, F(a^1))$ is the following set

$$\begin{aligned}\{(a^3, F(a^3)) \in (F, A) : \widetilde{G}((a^3, F(a^3)), (a^1, F(a^1)), (a^1, F(a^1)))\\ \preceq (\widetilde{\pi}(a^3, a^1, a^1), \widetilde{\pi}(a^3, a^1, a^1))\\ = (a'^1, r)\}.\end{aligned}$$

We show a soft ball with a radius of (a'^1, r) around $(a^1, F(a^1))$ by $\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r))$, And if there is no ambiguity about the soft metric $\widetilde{G}$, we display the said ball as $\mathcal{B}((a^1, F(a^1)), (a'^1, r))$ or $\mathcal{B}_{(a'^1,r)}(a^1, F(a^1))$.

Theorem 1 *Consider the soft set (F, A) on (X, d), let $\widetilde{G}$ be a soft G-metric on (F, A). So, the collection*

$$\mathcal{B} = \{\mathcal{B}_{\widetilde{G}}(a^1, F(a^1)), (a'^1, r)) : (a^1, F(a^1)) \in (F, A), a'^1 \in A, r \in \mathbb{R}^+\}.$$

is a base for (F, A).

Definition 9 Consider the soft set (F, A) on X. A soft sequence in (F, A) is a function $f : \mathbb{N} \longrightarrow (F, A)$ by setting $f(n) = (F_n, A),\ n \in \mathbb{N}$, such that (F_n, A) is a soft subset of (F, A) for $n \in \mathbb{N}$, and we show it by $\{(F_n, A)\}_{n=1}^{\infty}$.

Definition 10 Consider the soft set (F, A) on X, let $\widetilde{G}$ be a soft G-metric on (F, A), $\{(F_n, A)\}_{n=1}^{\infty}$ be a soft sequence in (F, A) and $(x^1, F(x^1)) \in (F, A)$. Then we call $\{(F_n, A)\}_{n=1}^{\infty}$ G-converges to $(x^1, F(x^1))$, if for every number $\epsilon > 0$, there exists a natural number N such that for every $n, m \in \mathbb{N}$ which $n, m \geq \mathbb{N}$, we have

$$\widetilde{G}((a^1, F_n(a^1)), (a^2, F_m(a^2)), (x^1, F(x^1))) \preceq (\widetilde{\pi}(a^1, a^2, x^1), \widetilde{\pi}(a^1, a^2, x^1), \epsilon).$$

In other words, $\{(F_n, A)\}_{n=1}^{\infty}$ G-converges to $(x^1, F(x^1))$, if for every positive number ϵ such that for every $n \geq N$, $(a^1, F_n(a^1))$ lies in soft ball of radius $(\widetilde{\pi}(a^1, x^1), \widetilde{\pi}(a^2, x^1), \epsilon)$ about $(x^1, F(x^1))$,

$$\mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(a^1, a^2, x^1), \widetilde{\pi}(a^1, a^2, x^1), \epsilon))).$$

The meaning of $(F_n, A) \longrightarrow (x^1, F(x^1))$, means sequence $\{(F_n, A)\}_{n=1}^{\infty}$ G-converging to $(x^1, F(x^1))$, for all $n \in \mathbb{N}$.

Lemma 1 *Every soft G-metric space is a Hausdorff soft space.*

Theorem 2 *Consider the soft set (F, A) on X, let $\widetilde{G}$ be a metric on (F, A), $\{(F_n, A)\}_{n=1}^{\infty}$ be a soft sequence in (F, A). If $\{(F_n, A)\}_{n=1}^{\infty}$ is G-convergent in (F, A), then it G-converges to unique element of (F, A).*

Proof (reductio ad absurdum), let there are elements

$$(x^1, F(x^1)), (x^2, F(x^2), (x^3, F(x^3)) \in (F, A)$$

such that $(F_n, A) \longrightarrow (x^1, F(x^1))$, $(F_n, A) \longrightarrow (x^2, F(x^2))$ and $(F_n, A) \longrightarrow (x^3, F(x^3))$, for every $n \in \mathbb{N}$. Then by condition 3. of definition 5, for every $a^1 \in A$, we have

$$\begin{aligned}\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)),&(x^3, F(x^3)) \\ &\preceq \widetilde{G}((x^1, F(x^1)), (a^1, F_n(a^1)), (a^1, F_n(a^1))) \\ &\dotplus \widetilde{G}((a^1, F_n(a^1)), (x^2, F(x^2)), (x^3, F(x^3))). \quad (1)\end{aligned}$$

Now, let ϵ be an arbitrary positive number. Since $\{(F_n, A)\}_{n=1}^{\infty}$ G-converges to $(x^1, F(x^1))$, $(x^2, F(x^2))$ and $(x^3, F(x^3))$, so there exist numbers $N_1, N_2 \in \mathbb{N}$ such that for every $n > N_1$ and $n > N_2$,

$$\begin{aligned}\widetilde{G}((x^1, F(x^1)), (a^1, F_n(a^1)),&(a^1, F_n(a^1))) \\ &\preceq (\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(x^1, a^1, a^1), \epsilon/2), \quad (2)\end{aligned}$$

and

$$\begin{aligned}\widetilde{G}((a^1, F_n(a^1)), (x^2, F(x^2)),&(x^3, F(x^3))) \\ &\preceq (\widetilde{\pi}(a^1, x^2, x^3), \widetilde{\pi}(a^1, x^2, x^3), \epsilon/2). \quad (3)\end{aligned}$$

Now, suppose that $n \geq \max\{N_1, N_2\}$. Then by (1), (2) and (3), we have

$$\begin{aligned}\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)),&(x^3, F(x^3)) \\ &\preceq (\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(x^1, a^1, a^1), \epsilon/2) \\ &\quad \dotplus (\widetilde{\pi}(a^1, x^2, x^3), \widetilde{\pi}(a^1, x^2, x^3), \epsilon/2) \\ &\preceq (\{\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(a^1, x^2, x^3)\}, \\ &\quad \{\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(a^1, x^2, x^3)\}, \epsilon). \quad (4)\end{aligned}$$

Therefore

$$\begin{aligned}\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)), (x^3, F(x^3)) \preceq \Big(&\{\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(a^1, x^2, x^3)\}, \\ &\{\widetilde{\pi}(x^1, a^1, a^1), \widetilde{\pi}(a^1, x^2, x^3)\}, \epsilon\Big),\end{aligned}$$

for every $\epsilon > 0$ and $a^1 \in A$. Then

$$\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)), (x^3, F(x^3))) = (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), 0).$$

So condition (1) of definition 5, implies that $x^1 = x^2 = x^3$, and consequently, $(x^1, F(x^1)) = (x^2, F(x^2)) = (x^3, F(x^3))$.

Definition 11 Consider the soft set (F, A) on X, let $\widetilde{G}$ be a metric on (F, A), $\{(F_n, A)\}_{n=1}^{\infty}$ be a soft sequence in (F, A). Then we call $\{(F_n, A)\}_{n=1}^{\infty}$ is a G-Cauchy soft sequence, if for every number $\epsilon > 0$, there is a natural number $\mathbb{N}$ such that for every $n, m \in \mathbb{N}$ that $n, m \geq \mathbb{N}$, we have

$$\widetilde{G}((a^1, F_n(a^1)), (a^1, F_m(a^1)),(a^1, F_m(a^1))) \\ \preceq (\widetilde{\pi}(a^1, a^1, a^1), \widetilde{\pi}(a^1, a^1, a^1), \epsilon).$$

Proposition 1 *Let $((F, A), \widetilde{G})$ be a soft G-metric space on X and let $\{(F_n, A)\}_{n=1}^{\infty}$ be a G-convergent soft sequence in (F, A). Then $\{(F_n, A)\}_{n=1}^{\infty}$ is a G-Cauchy soft sequence.*

Proof It is Straightforward.

Definition 12 Consider the soft set (F, A) on X, let $\widetilde{G}$ be a metric on (F, A). We call (F, A) is a complete soft G-metric space if every G-Cauchy soft sequence G-converges in (F, A).

Let $((F, A), \widetilde{G})$ and $((F', A'), \widetilde{G}')$ be two soft G-metric spaces on X and Y, respectively. Using the definition 5, we define the mappings $\widetilde{\pi} : A \times A \longrightarrow A$ and $\widetilde{\pi}' : A' \times A' \longrightarrow A'$. Now, we consider continuity of soft mappings on soft G-metric spaces.

Theorem 3 *Let $((F, A), \widetilde{G})$ and $((F', A'), \widetilde{G}')$ be two soft G-metric spaces on X and Y, respectively. Let $f = (f_1, f_2) : ((F, A), \widetilde{G}) \longrightarrow ((F', A'), \widetilde{G}')$ be a soft mapping. Then f is soft continuous if and only if for every $(x^1, F(x^1)) \in (F, A)$ and every $\epsilon > 0$, there are $\delta > 0$ such that for every $(x^2, F(x^2)), (x^3, F(x^3)) \in (F, A)$*

$$\widetilde{G}'(f(x^1, F(x^1)), f(x^2, F(x^2)),f(x^3, F(x^3))) \\ \preceq (\widetilde{\pi}'(\widetilde{\pi}(x^1, x^2, x^3)), \widetilde{\pi}'(\widetilde{\pi}(x^1, x^2, x^3)), \epsilon), \quad (5)$$

whenever

$$\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)),(x^3, F(x^3))) \\ \preceq (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta).$$

Proof Let $f : ((F, A), \widetilde{G}) \longrightarrow ((F', A'), \widetilde{G}')$ be soft continuous, $\epsilon > 0$ and $(x^1, F(x^1)) \in (F, A)$. By Theorem 1, the soft ball

$$\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \epsilon)))$$

in $((F', A'), \widetilde{G}')$ is open, for all $u, v \in A$. Since f is soft continuous, then

$$f^{-1}(\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v)), \epsilon)))$$

in $((F, A), \widetilde{G})$ is open. Condition 3. of Theorem 2.3 of [6], implies that

$$(x^1, F(x^1)) \\ \in f^{-1}(\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \epsilon))).$$

Then there is a positive number δ such that

$$\begin{aligned}&(x^1, F(x^1))\\ &\quad \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, u, v), \widetilde{\pi}(x^1, u, v), \delta))\\ &\quad \widetilde{\subset} f^{-1}(\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \epsilon))),\end{aligned}$$

for all $u, v \in A$. If $x^2, x^3 \in A$ is an element with

$$\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)), (x^3, F(x^3))) \preceq (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta),$$

then

$$(x^2, F(x^2)), (x^3, F(x^3)) \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta)).$$

This shows that

$$\begin{aligned}&(x^2, F(x^2)), (x^3, F(x^3)) \in\\ &f^{-1}(\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon))).\end{aligned}$$

Now, using condition 4, theorem 2.3 of [6], we conclude that $f((x^2, F(x^2)), f((x^3, F(x^3))$ belongs to

$$\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon))).$$

Therefore (5) holds.

Converse, assume that (W, C) in $((F', A'), \widetilde{G}')$ be an open set. Suppose that $(x^1, F(x^1)) \in f^{-1}((W, C))$. Again by Theorem 2.3 of [6], we have $f(x^1, F(x^1)) \in (W, C)$. Since (W, C) is open in $((F', A'), \widetilde{G}')$, then by Theorem 1, there exist a number $\epsilon > 0$ and soft ball

$$\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \epsilon))$$

in $((F', A'), \widetilde{G}')$, for all $u, v \in A$ such that

$$\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, u, v))), \epsilon)))\widetilde{\subset}(W, C). \quad (6)$$

On the other hand, there is a positive number δ such that for every $(x^2, F(x^2)), (x^3, F(x^3)) \in (F, A)$

$$\begin{aligned}&\widetilde{G}'(f((x^1, F(x^1))), f((x^2, F(x^2))), f((x^3, F(x^3))))\\ &\qquad \preceq (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon), \qquad (7)\end{aligned}$$

whenever

$$\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)), (x^3, F(x^3))) \preceq (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta).$$

As a result for every $(x^2, F(x^2)), (x^3, F(x^3)) \in (F, A)$, if

$$\begin{aligned}&(x^2, F(x^2)), (x^3, F(x^3))\\ &\qquad \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta))), \qquad (8)\end{aligned}$$

then

$$\begin{aligned}&f(x^2, F(x^2)), f(x^3, F(x^3))\\&\quad\in \mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon))).\end{aligned} \tag{9}$$

Therefore

$$\begin{aligned}&f(\mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta)))\\&\quad\widetilde{\subset}\mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon)).\end{aligned} \tag{10}$$

Using (6) and (10), we have

$$f(\mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta)))\widetilde{\subset}(W, C).$$

Then

$$(x^1, F(x^1)) \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta))\widetilde{\subset} f^{-1}(W, C).$$

This means $f^{-1}(W, C)$ in $((F, A), \widetilde{G})$ is an open soft set.

Lemma 2 *Let $((F, A), \widetilde{G})$ be a soft G-metric spaces on X and (S, B) be a soft subset of (F, A). Then $(x^1, S(x^1)) \in \overline{(S, B)}$ if and only if there is a soft sequence $\{(S_n, B)\}_{n=1}^{\infty}$ in (S, B) such that $\{(S_n, B)\}_{n=1}^{\infty}$ G-converges to $(x^1, S(x^1))$.*

Proof According to the Theorem 5 from [9], we have $\overline{(S, B)} = (S, B)^{\circ}\widetilde{\cup}\underline{(S, B)}$. So, if $(x^1, S(x^1)) \in \overline{(S, B)}$ so it is in $(S, B)^{\circ}$ or $\underline{(S, B)}$. If $(x^1, S(x^1)) \in (S, B)^{\circ}$, suffice that is to suppose that $(S_1, A) = (x^1, S(x^1))$ and $(S_n, A) = \widetilde{\emptyset}$ for $n \geq 2$. Now assume that $(x^1, S(x^1)) \in \underline{(S, B)}$. Since $\underline{(S, B)} = \overline{(S, B)}\widetilde{\cap}\overline{((S, B)^{a^3})}$, then there exist a

$$(x_1^1, S(x_1^1)) \in \mathcal{B}_{\widetilde{G}}((x^1, S(x^1)), (\widetilde{\pi}(x^1, x_1^1, x_1^1), \widetilde{\pi}(x^1, x_1^1, x_1^1), 1))\widetilde{\cap}\overline{((S, B)^{a^3})},$$

such that $x_1^1 \neq x^1$. Thus

$$\widetilde{G}((x^1, S(x^1)), (x_1^1, S(x_1^1)), (x_1^1, S(x_1^1))) \preceq (\widetilde{\pi}(x^1, x_1^1, x_1^1), \widetilde{\pi}(x^1, x_1^1, x_1^1), 1).$$

Now, let $\epsilon > 0$ and

$$\begin{aligned}&(\widetilde{\pi}(x^1, x_1^1, x_1^1), \widetilde{\pi}(x^1, x_1^1, x_1^1), \epsilon_1)\\&\qquad= \frac{1}{2}\widetilde{G}((x^1, S(x^1)), (x_1^1, S(x_1^1)), (x_1^1, S(x_1^1))).\end{aligned}$$

We have $(x^1, S(x^1)) \in \underline{(S, B)}$, then there exist a $(x_2^1, S(x_2^1)) \neq (x^1, S(x^1))$ belongs to

$$\mathcal{B}_{\widetilde{G}}((x^1, S(x^1)), (\widetilde{\pi}(x^1, x_2^1, x_2^1), \widetilde{\pi}(x^1, x_2^1, x_2^1), \epsilon_1))\widetilde{\cap}\overline{((S, B)^{a^3})}. \tag{11}$$

Then (11) implies that

$$\widetilde{G}((x^1, S(x^1)), (x_2^1, S(x_2^1)), (x_2^1, S(x_2^1))) \preceq (\widetilde{\pi}(x^1, x_2^1, x_2^1), \widetilde{\pi}(x^1, x_2^1, x_2^1), \epsilon_1),$$

and clearly, $x_2^1 \neq x_1^1$. Now, let

$$(\widetilde{\pi}(x^1, x_2^1, x_2^1), \widetilde{\pi}(x^1, x_2^1, x_2^1), \epsilon_2) = \frac{1}{2}\widetilde{G}((x^1, S(x^1)), (x_2^1, S(x_2^1)), (x_2^1, S(x_2^1))).$$

By induction, we construct an infinite soft sequence with separate elements of (S, B), and we set $(S_n, B) = (x_n^1, S(x_n^1))$ for all $n = 1, 2, \ldots$. Hence

$$\begin{aligned}\widetilde{G}((S_n, B), (x^1, S(x^1)), &(x^1, S(x^1)))\\ &\preceq (\widetilde{\pi}(x^1, x_n^1, x_n^1), \widetilde{\pi}(x^1, x_n^1, x_n^1), \epsilon_{n-1})\\ &\preceq (\widetilde{\pi}(x^1, x_n^1, x_n^1), \widetilde{\pi}(x^1, x_n^1, x_n^1), \frac{1}{2}\epsilon_{n-2})\\ &\preceq \cdots \preceq (\widetilde{\pi}(x^1, x_n^1, x_n^1), \widetilde{\pi}(x^1, x_n^1, x_n^1), \frac{1}{2^{n-2}}\epsilon_1)\\ &\preceq (\widetilde{\pi}(x^1, x_n^1, x_n^1), \widetilde{\pi}(x^1, x_n^1, x_n^1), \frac{1}{2^{n-1}}).\end{aligned}$$

Therefor, $(S_n, B) \longrightarrow (x^1, S(x^1))$ as $n \longrightarrow \infty$. The inverse is clear.

Theorem 4 *Let $((F, A), \widetilde{G})$ and $((F', A'), \widetilde{G}')$ be two soft G-metric spaces on X and Y, respectively. Let $f = (f_1, f_2) : ((F, A), \widetilde{G}) \longrightarrow ((F', A'), \widetilde{G}')$ be a soft mapping. Therefore f is soft continuous if and only if for every G-convergent soft sequence $\{(F_n, A)\}_{n=1}^{\infty}$ in (F, A) which G-converges to $(x^1, F(x^1))$ in (F, A), the sequence $\{f((F_n, A))\}_{n=1}^{\infty}$ G-converges to $f((x^1, F(x^1)))$.*

Proof Let $f : ((F, A), \widetilde{G}) \longrightarrow ((F', A'), \widetilde{G}')$ be soft continuous, and $\{(F_n, A)\}_{n=1}^{\infty}$ be a soft sequence in (F, A) such that $\{(F_n, A)\}_{n=1}^{\infty}$ is G-converges to $(x^1, F(x^1)) \in (F, A)$. Let $\epsilon > 0$. Since f is soft continuous, then by Theorem 3, there is $\delta > 0$ such that for every $(x^2, F(x^2)), (x^3, F(x^3)) \in (F, A)$,

$$\begin{aligned}&f((x^2, F(x^2))), f((x^3, F(x^3)))\\ &\in \mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon))),\end{aligned} \quad (12)$$

whenever

$$(x^2, F(x^2)), (x^3, F(x^3)) \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta))).$$

On the other hand, there is a positive integer $N \in \mathbb{N}$ such that for $n \geq N$, $(F_n, A) \in \mathcal{B}_{\widetilde{G}}((x^1, F(x^1)), (\widetilde{\pi}(x^1, x^2, x^3), \widetilde{\pi}(x^1, x^2, x^3), \delta))$. Therefore, we conclude that for every $n \geq N$,

$$\begin{aligned}&f((F_n, A))\\ &\in \mathcal{B}_{\widetilde{G}'}(f((x^1, F(x^1))), (\widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \widetilde{\pi}'(f_1(\widetilde{\pi}(x^1, x^2, x^3))), \epsilon))).\end{aligned} \quad (13)$$

Therefore $f((F_n, A))$, G-converges to $f((x^1, F(x^1))$.

Assuming that for every G-convergent soft sequence in (F, A) such as $\{(F_n, A)\}_{n=1}^{\infty}$ which G-converges to any $(x^1, F(x^1))$ in (F, A), the sequence

$\{f((F_n,A))\}_{n=1}^{\infty}$, G-converges to $f((x^1,F(x^1)))$. Let (S,B) be a soft subset in (F,A) and let $(x^1,S(x^1))\in\overline{(S,B)}$. So by Lemma 2, there is a soft sequence $\{(S_n,B)\}_{n=1}^{\infty}$ in (S,B) such that $\{(S_n,B)\}_{n=1}^{\infty}$ G-converges to $(x^1,S(x^1))$. Then our assumption concludes that
$f((S_n,B))$ G-converges to $f((x^1,G(x^1)))$.
Therefore $\underline{f((S_n,B))}\in f(S,B)$, as a result $f((x^1,S(x^1)))\in\overline{f(S,B)}$. That is, $f(\overline{(S,B)})\widetilde{\subset}\overline{f((S,B))}$. Now, by using Theorem 4.2 of [7], f is soft continuous.

2 Banach Contraction Theorem

In this section, we prove the Banach contraction theorem for soft G-metric spaces. First, we have the following definition.

Definition 13 Let $((F,A),\widetilde{G})$ be a soft G-metric space on X, and $f:((F,A)\widetilde{G})\to((F,A),\widetilde{G})$ be a soft mapping. We say f is soft G-contractive if there exist a positive number c with $0<c<1$ such that

$$\begin{aligned}&\widetilde{G}(f((x^1,F(x^1))),f((x^2,F(x^2))),f((x^3,F(x^3))))\\&\quad\preceq c\Big[\widetilde{G}\Big((x^1,F(x^1),F(x^1)),(x^2,F(x^2),F(x^2)),(x^3,F(x^3),F(x^3))\Big)\Big],\end{aligned}$$

for all $x^1,x^2,x^3\in A$.

Clearly, by Theorem 3, any soft contractive mapping will be soft continuous.

Definition 14 Consider the soft metric space $((F,A),\widetilde{G})$ on X, and let $f:((F,A),\widetilde{G})\to((F,A),\widetilde{G})$ be a soft mapping. A fixed soft set for f is a soft subset of (F,A) such as $(x^1,F(x^1))$ such that $f((x^1,F(x^1)))=(x^1,F(x^1))$.

Theorem 5 *Let $((F,A),\widetilde{G})$ be a complete soft G-metric space on X, and let $f:((F,A),\widetilde{G})\longrightarrow((F,A),\widetilde{G})$ be a soft contractive mapping. Threfore f has a unique fixed soft set.*

Proof Let (F_0,A) be an arbitrary soft point in (F,A). We make soft sequence $\{(F_n,A)\}_{n=1}^{\infty}$ as follows:

$$(F_{n+1},A)=f((F_n,A))\qquad n=0,1,2,\ldots. \tag{14}$$

Since f is soft contraction, there exists $0<c<1$ such that for all $n\geq 1$

$$\begin{aligned}&\widetilde{G}((F_n,A),(F_{n+1},A),(F_{n+1},A))\\&\qquad=\widetilde{G}(f((F_{n-1},A)),f((F_n,A)),f((F_n,A)))\\&\qquad\leq c\widetilde{G}\left((F_{n-1},A),(F_n,A),(F_n,A)\right)\\&\qquad\vdots\\&\qquad\leq c^n\widetilde{G}\left((F_0,A),(F_1,A),(F_1,A)\right).\end{aligned}\tag{15}$$

For any $n < m$, by repeated use of condition (3) in Definition 5, we have

$$\begin{aligned}
&\widetilde{G}\Big((F_n, A), (F_m, A), (F_m, A)\Big) \\
&\leq \widetilde{G}\Big((F_n, A), (F_{n+1}, A), (F_{n+1}, A)\Big) \\
&\dot{+} \cdots \dot{+} [\widetilde{G}\Big((F_{m-1}, A), (F_m, A), (F_m, A)\Big)] \\
&\leq c^n[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)] \\
&\dot{+} c^{n+1}[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)] \\
&\dot{+} \cdots \dot{+} c^{m-1}[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)] \\
&= (\sum_{n=0}^{m-1} c^n)[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)] \\
&\leq (\sum_{n=0}^{\infty} c^n)[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)] \\
&= \frac{1}{1-a^3}[\widetilde{G}\Big((F_0, A), (F_1, A), (F_1, A)\Big)]. \quad (16)
\end{aligned}$$

Thus, (16) follows that $\{(F_n, A)\}_{n=1}^{\infty}$ is a G-Cauchy soft sequence in (F, A). Since (F, A) is a complete soft G-metric space, then there is $(x^1, F(x^1)) \in (F, A)$ such that $\{(F_n, A)\}_{n=1}^{\infty}$ G-converges to $(x^1, F(x^1))$. Since f is soft continuous, then by Theorem 4, $f((F_n, A))$ G-converges to $f((x^1, F(x^1))$. Also by definition of $\{(F_n, A)\}_{n=1}^{\infty}$, $f((F_n, A))$ G-converges to $(x^1, F(x^1))$. Thus using the theorem 2 $f((x^1, F(x^1))) = (x^1, F(x^1))$.

To prove uniqueness of $(x^1, F(x^1))$, suppose there is another element $(x^2, F(x^2)) \in (F, A)$ such that $f((x^2, F(x^2))) = (x^2, F(x^2))$ and
$(x^1, F(x^1)) \neq (x^2, F(x^2))$. Then

$$\begin{aligned}
&\widetilde{G}((x^1, F(x^1)), (x^2, F(x^2)), (x^2, F(x^2))) \\
&= \widetilde{G}(f((x^1, F(x^1))), f((x^2, F(x^2))), f((x^2, F(x^2)))) \\
&\leq c[\widetilde{G}\Big((x^1, F(x^1)), (x^2, F(x^2)), (x^2, F(x^2))\Big)].
\end{aligned}$$

Thus the above inequality conclude that $c > 1$, so this would be a contradiction, as a result, the proof is complete.

Example 1 Let $X = [0, \infty]$, $d(x^1, x^2) = |x^1 - x^2|$, $A = \mathbb{Q}^+ = \{x^1 \in \mathbb{Q} : x^1 > 0\}$ and let (F, A) be a soft set on X such that each $(a, F(a^1)) \in (F, A)$ is defined as $(a^1, F(a^1)) = [0, a^1]$. Consider the mapping $\widetilde{G} : (F, A) \times (F, A) \times (F, A) \longrightarrow (A, \mathbb{R}^+ \cup \{0\})$ defined by

$$\begin{aligned}
&\widetilde{G}((a^1, F(a^1)), (a^2, F(a^2)), (a'^3, F(a'^3))) \\
&= \max\{|a^1 - a^2|, |a^2 - a'^3|, |a'^3 - a^1|\}.
\end{aligned}$$

Clearly, $((F,A),\widetilde{G})$ is a complete soft G-metric space on X. Suppose f : $((F,A),\widetilde{G}) \longrightarrow ((F,A),\widetilde{G})$ be a soft mapping with the following definition

$$f((a^1,F(a^1))) = (\frac{1}{4}a^1, F(\frac{1}{4}a^1)) = [0,\frac{1}{4}a^1].$$

Then,

$$\begin{aligned}
&\widetilde{G}(f((a^1,F(a^1))),f((a^2,F(a^2))),f((a'^3,F(a'^3))))\\
&\qquad = \widetilde{G}((\frac{1}{4}a^1,F(\frac{1}{4}a^1)),(\frac{1}{4}a^2,F(\frac{1}{4}a^2)),(\frac{1}{4}a'^3,F(\frac{1}{4}a'^3)))\\
&\qquad = \max\{|\frac{1}{4}a^1-\frac{1}{4}a^2|,|\frac{1}{4}a^2-\frac{1}{4}a'^3|,|\frac{1}{4}a'^3-\frac{1}{4}a^1|)\\
&\qquad = \frac{1}{2}\max\{|\frac{1}{2}|a^1-a^2|,\frac{1}{2}|a^2-a'^3|,|\frac{1}{2}|a'^3-a^1|)\\
&\qquad \le \frac{1}{2}\max\{|a^1-a^2|,|a^2-a'^3|,|a'^3-a^1|)\\
&\qquad = \frac{1}{2}\widetilde{G}((a^1,F(a^1)),(a^2,F(a^2)),(a'^3,F(a'^3))).
\end{aligned}$$

The above relations show that f is a contractive soft map with $c=\frac{1}{2}$. Now by applying the theorem 5 we conclude that f has a unique fixed soft set.

Theorem 6 *Let* $((F,A),\widetilde{G})$ *be a complete soft* G*-metric space on* X*, and* $T:(F,A)\longrightarrow(F,A)$ *be a soft mapping such that* T^m *satisfies the following conditions:*

$$\begin{aligned}
&\widetilde{G}\Big(T^m((a^1,F(a^1))),T^m((a^2,F(a^2))),T^m((a^2,F(a^2)))\Big)\\
&\qquad \preceq c\Big[\widetilde{G}\Big((a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\Big)\Big], \qquad (17)
\end{aligned}$$

for every $(a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\in(F,A)$ *and* $0<c<1$*.Then* T *has a unique fixed soft set.*

Proof According to the theorem 5, T^m has a unique fixed soft set $(a^1,F(a^1))$. However,

$$T^{m+1}(a^1,F(a^1)) = T(T^m(a^1,F(a^1))) = T(a^1,F(a^1));$$

therefore, $T(a^1,F(a^1))$ is also a fixed point of T^m. Since the fixed soft set of T^m is unique, so, as a result $T(a^1,F(a^1))=(a^1,F(a^1))$. For uniqueness, if $T(a^2,F(a^2))=(a^2,F(a^2))$ then $T^n(a^2,F(a^2))=(a^2,F(a^2))$ which prove that $(a^2,F(a^2))=(a^1,F(a^1))$.

Example 2 As in Example 1, let $X=[0,\infty]$, $d(x^1,x^2)=|x^1-x^2|$, $A=\mathbb{Q}^+=\{x^1\in\mathbb{Q}:x^1>0\}$ and (F,A) be a soft set on X such that each $(a^1,F(a^1))\in(F,A)$ defined as $(a^1,F(a^1))=[0,a^1]$. Consider the mapping $\widetilde{G}:(F,A)\times(F,A)\times(F,A)\longrightarrow(A,\mathbb{R}^+\cup\{0\})$ defined by $\widetilde{G}((a^1,F(a^1)),(a^2,F(a^2)),(a'^3,F(a'^3)))=$

$\max\{|a^1 - a^2|, |a^2 - a'^3|, |a'^3 - a^1|)$. Let $T : ((F, A), \widetilde{G}) \longrightarrow ((F, A), \widetilde{G})$ be a soft function defined by

$$T((a^1, F(a^1))) = (\frac{1}{4}a^1, F(\frac{1}{4}a^1)) = [0, \frac{1}{4}a^1].$$

By definition of T we have

$$\begin{aligned} T^2((a^1, F(a^1)) = T(T((a^1, F(a^1)))) &= T((\frac{1}{4}a^1, F(\frac{1}{4}a^1)) \\ &= (\frac{1}{4^2}a^1, F(\frac{1}{4^2}a^1)). \end{aligned}$$

Then by induction, $T^m((a^1, F(a^1))) = (\frac{1}{4^m}a^1, F(\frac{1}{4^m}a^1))$, for $m \in \mathbb{N}$. Thus,

$$\begin{aligned} &\widetilde{G}(T^m((a^1, F(a^1))), T^m((a^2, F(a^2))), T^m((a'^3, F(a'^3)))) \\ &= \widetilde{G}((\frac{1}{4^m}a^1, F(\frac{1}{4^m}a^1)), (\frac{1}{4^m}a^2, F(\frac{1}{4^m}a^2)), (\frac{1}{4^m}a'^3, F(\frac{1}{4^m}a'^3))) \\ &= \max\{|\frac{1}{4^m}a^1 - \frac{1}{4^m}a^2|, |\frac{1}{4^m}a^2 - \frac{1}{4^m}a'^3|, |\frac{1}{4^m}a'^3 - \frac{1}{4^m}a^1|) \\ &= \frac{1}{4}\max\{\frac{1}{4^{m-1}}|a^1 - a^2|, \frac{1}{4^{m-1}}|a^2 - a'^3|, \frac{1}{4^{m-1}}|a'^3 - a^1|) \\ &\preceq \frac{1}{4}\max\{|a^1 - a^2|, |a^2 - a'^3|, |a'^3 - a^1|) \\ &= \frac{1}{4}\widetilde{G}((a^1, F(a^1)), (a^2, F(a^2)), (a'^3, F(a'^3))). \end{aligned}$$

Theorem 7 *Let $((F, A), \widetilde{G})$ be a complete soft G-metric space on X, and let $T : \mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r)) \longrightarrow ((F, A), \widetilde{G})$ be a soft contraction mapping with*

$$\widetilde{G}\Big(T((a^1, F(a^1)), (a^1, F(a^1), (a^1, F(a^1))\Big) \prec (1 - c)(a'^1, r), \tag{18}$$

where $0 < c < 1$. Then T has a unique fixed soft set in

$$\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r)).$$

Proof Clearly, there is an $r_0 \geq 0$ such that $0 \preceq (a'^1, r_0) \preceq (a'^1, r)$ with

$$\widetilde{G}\Big(T((a^1, F(a^1)), (a^1, F(a^1), (a^1, F(a^1))\Big) \preceq (1 - c)(a'^1, r_0).$$

Now; take $T : \overline{\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r_0))} \to \overline{\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r_0))}$. If $(a^2, F(a^2)) \in \overline{\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r_0))}$, then

$$\begin{aligned}
\widetilde{G}\Big(T((a^2, F(a^2))),&(a^1, F(a^1)), (a^1, F(a^1))\Big) \\
&\preceq \widetilde{G}\Big(T((a^2, F(a^2))), T((a^1, F(a^1))), T((a^1, F(a^1)))\Big) \\
&+ \widetilde{G}\Big(T((a^1, F(a^1)), T((a^1, F(a^1))), (a^1, F(a^1))\Big) \\
&\preceq c\widetilde{G}\Big((a^2, F(a^2)), (a^1, F(a^1)), (a^1, F(a^1))\Big) \\
&+ (1-c)(a'^1, r_0) \\
&\preceq c(a'^1, r_0) + (1-c)(a'^1, r_0) \\
&= (a'^1, r_0).
\end{aligned}$$

Since $\overline{\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r_0))} \widetilde{\subseteq} \mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r))$. Now, by applying Theorem 5 we conclude that T has a unique fixed soft set in

$$\overline{\mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r_0))} \widetilde{\subseteq} \mathcal{B}_{\widetilde{G}}((a^1, F(a^1)), (a'^1, r)).$$

3 Extend some fixed point theorems

Lemma 3 *Let $A \subseteq E$ be a set of parameters, (a^1, r) and (a^2, r') be two soft parametric scalars such that for every $\epsilon > 0$, if $(a^1, r) \prec (a^2, r' + \epsilon)$, then $(a^1, r) \preceq (a^2, r')$.*

Theorem 8 *Let $((F, A), \widetilde{G})$ be a complete soft G-metric space on X, and let $T = (T_1, T_2) : ((F, A), \widetilde{G}) \longrightarrow ((F, A), \widetilde{G})$ be a soft continuous mapping such that*

$$T((a^1, F(a^1)) = (T_1(a^1), T_2(F(a^1)) = (T_1(a^1), F(T_1(a^1)))$$

for every $(a^1, F(a^1)) \in (F, A)$ and it satisfies for some parametric scaler valued $\varphi : (F, A) \longrightarrow (A, \mathbb{R}^+)$

$$\begin{aligned}
\widetilde{G}\Big(T((a^1, F(a^1)), (a^1, F(a^1)),&(a^1, F(a^1))\Big) \\
&\prec \varphi((a^1, F(a^1))) - \varphi(T(a^1, F(a^1))). \qquad (19)
\end{aligned}$$

Then $\{T^n((a^1, F(a^1))\}$ converges to a fixed soft set, for every $(a^1, F(a^1)) \in (F, A)$.

Proof By definitions 1 and 3, set $T = (T_1, T_2)$ and $\varphi = (\varphi_1, \varphi_2)$. Now, if $T((a^1, F(a^1))) = (T_1(a^1), T_2(F(a^1)))$, we set $T((a^1, F(a^1))) = (a^1_1, F(a^1_1))$, then

$$\varphi(T((a^1, F(a^1)))) = (\varphi_1(a^1_1), \varphi_2(F(a^1_1))) = (a^2_1, r_1).$$

Similarly, we write $T^n((a^1, F(a^1))) = (a^1_n, F(a^1_n))$ for $n = 1, 2, \ldots$. Thus

$$\varphi(T^n((a^1, F(a^1)))) = (\varphi_1(a^1_n), \varphi_2(F(a^1_n))) = (a^2_n, r_n).$$

From the condition (22) we have

$$\widetilde{G}\Big(T((a^1,F(a^1)),(a^1,F(a^1)),(a^1,F(a^1))\Big)+\varphi(T(a^1,F(a^1))) \prec \varphi((a^1,F(a^1)). \tag{20}$$

So by Lemma 3, $\varphi(T((a^1,F(a^1)))) \preceq \varphi((a^1,F(a^1))$. This implies that $\varphi(T^2((a^1,F(a^1))) \preceq \varphi(T((a^1,F(a^1)))$. By continuing this process, we obtain $\varphi(T^{n+1}((a^1,F(a^1))) \preceq \varphi(T^n((a^1,F(a^1))))$. That is, $\{\varphi(T^n((a^1,F(a^1))))\}$ is a decreasing and as a result the sequence $\{r_n\}$ of real numbers is decreasing. Hence there exist $r \in \mathbb{R}$ such that

$$\lim_{n\longrightarrow\infty}\varphi(T^n((a^1,F(a^1)))) = (a^2,r) \qquad \text{(for all } a^2 \in A). \tag{21}$$

Clearly (a^2,r) is nonnegative. Then for all $n,m \in \mathbb{N}$ with $n \le m$, we have

$$\begin{aligned}
&\widetilde{G}\Big(T^n((a^1,F(a^1))),T^m((a^1,F(a^1))),T^m((a^1,F(a^1)))\Big) \preceq \\
&\quad \widetilde{G}\Big(T^n((a^1,F(a^1))),T^{n+1}((a^1,F(a^1))),T^{n+1}((a^1,F(a^1)))\Big) \\
&\qquad + \widetilde{G}\Big(T^{n+1}((a^1,F(a^1))),T^{n+2}((a^1,F(a^1))),T^{n+2}((a^1,F(a^1)))\Big) \\
&\quad \widetilde{G}\Big(T^{n+2}((a^1,F(a^1))),T^{n+3}((a^1,F(a^1))),T^{n+3}((a^1,F(a^1)))\Big) + \cdots \\
&\qquad + \widetilde{G}\Big(T^{m-1}((a^1,F(a^1))),T^{m}((a^1,F(a^1))),T^{m}((a^1,F(a^1)))\Big) \\
&\quad \preceq \sum_{i=n}^{m-1}\widetilde{G}\Big(T^i((a^1,F(a^1))),T^{i+1}((a^1,F(a^1))),T^{i+1}((a^1,F(a^1)))\Big) \\
&\quad \preceq \varphi(T^n((a^1,F(a^1)))) - \varphi(T^{n+1}((a^1,F(a^1)))) + \varphi(T^{n+1}((a^1,F(a^1)))) \\
&\qquad - \varphi(T^{n+2}((a^1,F(a^1))) + \cdots + \varphi(T^{m-1}((a^1,F(a^1)))) \\
&\qquad - \varphi(T^{m}((a^1,F(a^1)))) \\
&\quad = \varphi(T^n((a^1,F(a^1)))) - \varphi(T^m((a^1,F(a^1)))).
\end{aligned}$$

Thus,

$$\lim_{m.n\longrightarrow\infty}\widetilde{G}\Big(T^n((a^1,F(a^1))),T^m((a^1,F(a^1))),T^m((a^1,F(a^1)))\Big) = (a^2,0)$$

(see Definition 2). So, $\{T^n((a^1,F(a^1)))\}$ is a soft G-Cauchy sequence. Thus, there is $(x^1,F(x^1)) \in (F,A)$ such that

$$\lim_{n\longrightarrow\infty}T^n((a^1,F(a^1))) = (x^1,F(x^1)).$$

Definition 15 Suppose $((F,A),\widetilde{G})$ and $((S,B),\widetilde{G'})$ are soft G-metric spaces on X and Y, respectively, and $T:((F,A),\widetilde{G}) \to ((S,B),\widetilde{G'})$ be a soft mapping. We call T is a closed (open) soft map, if for every closed (open) soft set (F,A') in (F,A) where $A' \subseteq A$, $T((F,A'))$ is a closed (open) soft set in (S,B).

Some parts of the proof of the following theorem are similar to the proof of theorem 6.1 [11].

Theorem 9 *Let* $((F,A),\widetilde{G})$ *be a soft compact G-metric space on X, and* $T:((F,A),\widetilde{G})\longrightarrow((F,A),\widetilde{G})$ *be a soft closed mapping with the following condition*

$$\widetilde{G}\Big(T((a^1,F(a^1))),T((a^2,F(a^2))),T((a^3,F(a^3)))\Big) \prec \widetilde{G}\Big((a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\Big), \tag{22}$$

for every $(a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\in(F,A)$. *If for any nonempty soft set* $(a^1,F(a^1))$, $T((a^1,F(a^1))$ *is a nonempty soft set, then* T *has a unique fixed soft set in* (F,A).

Proof Let $\{A_n\}_{n\in\mathbb{N}}$ be a family of subsets of A, we put $T((F,A))=(F,A_1)$. similarly, set $T^2((F,A))=T((F,A_1))=(F,A_2)$. By continuing this process we have

$$T^3((F,A))=T((F,A_2))=(F,A_3),\cdots,T^n((F,A))=T((F,A_{n-1}))=(F,A_n),$$

for $n\in\mathbb{N}$. Clearly, $(F,A_{n+1})\widetilde{\subseteq}(F,A_n)$ for $n\in\mathbb{N}$. By Lemma 1 and Theorem 3.34 of [10], (F,A) is a soft closed set and so (F,A_1) is a soft closed set. Then for the same reason (F,A_n) is a soft closed set for $n\in\mathbb{N}$. As a result, by Proposition 6.1 of [11] we have $\widetilde{\cap}_{n\in\mathbb{N}}(F,A_n)\neq\widetilde{\emptyset}$. By Theorem 3.6 of [5] or Proposition 5.1 of [11] we have

$$\begin{aligned} T(\widetilde{\bigcap}_{n\in\mathbb{N}}(F,A_n)) &= T(\widetilde{\bigcap}_{n\in\mathbb{N}}T^n((F,A))) \\ &\widetilde{\subseteq}\widetilde{\bigcap}_{n\in\mathbb{N}}T^{n+1}((F,A)) \\ &\widetilde{\subseteq}\widetilde{\bigcap}_{n\in\mathbb{N}}T^{n}((F,A)) \\ &\widetilde{\subseteq}\widetilde{\bigcap}_{n\in\mathbb{N}}(F,A_n). \end{aligned}$$

Now we have to show that $\widetilde{\bigcap}_{n\in\mathbb{N}}(F,A_n)\widetilde{\subseteq}\, T(\widetilde{\bigcap}_{n\in\mathbb{N}}(F,A_n))$.
(reductio ad absurdum), Suppose there is a soft point $B\widetilde{\in}\bigcap_{n\in\mathbb{N}}(F,A_n)$ that is not a soft point in $T(\widetilde{\bigcap}_{n\in\mathbb{N}}(F,A_n))$.
Put $(F,B_n)=T^{-1}(B)\widetilde{\cap}(F,A_n)$. Then we have $(F,B_n)\widetilde{\subseteq}(F,B_{n-1})$ for every $n\in\mathbb{N}$. Again using the proposition 6.1 of [11], $\bigcap_{n\in\mathbb{N}}(F,B_n)\neq\widetilde{\emptyset}$. Then $T(\bigcap_{n\in\mathbb{N}}(F,B_n))\widetilde{\subseteq}B\widetilde{\cap}T(\bigcap_{n\in\mathbb{N}}(F,A_n))$. On the other hand since B is a soft point so $B=T(\bigcap_{n\in\mathbb{N}}(F,B_n))$. Thus, $B\in T(\bigcap_{n\in\mathbb{N}}(F,A_n))$. This completes the proof.

We put $\bigcap_{n\in\mathbb{N}}(F,A_n)=C$. Now we show that C is unique. Suppose there is another member such as $D\ in(F,A)$, such that $T(D)=D$ and $C\neq D$. Then

$$\widetilde{G}(C,D,D)=\widetilde{G}(T(C),T(D),T(D))\prec\widetilde{G}(C,D,D).$$

Thus, $\bigcap_{n\in\mathbb{N}}(F,A_n)$ is a unique fixed soft set.

Theorem 10 *Let $((F,A),\widetilde{G})$ be a complete soft G-metric space on X, and suppose there is α such that for every $(a^1,F(a^1))\in(F,A)$ soft mapping $T:(F,A)\longrightarrow(F,A)$ such that*

$$T((a^1,F(a^1))=(T_1(a^1),T_2(F(a^1))=(T_1(a^1),F(T_1(a^1)))$$

for every $(a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\in(F,A)$ and it satisfies:

$$\begin{aligned}\widetilde{G}\Big(T((a^1,F(a^1))),&T((a^2,F(a^2))),T((a^3,F(a^3)))\Big)\\&\preceq\alpha\Big[\widetilde{G}\Big((a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\Big)\Big]\\&\Big[\widetilde{G}\Big((a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\Big)\Big],\end{aligned}\tag{23}$$

for every $(a^1,F(a^1)),(a^2,F(a^2)),(a^3,F(a^3))\in(F,A)$, where $\alpha:(A,R^+)\to[0;1)$ satisfy to the following condition:

$$\alpha(a^1,t_n)\to 1\Rightarrow t_n\to 0.$$

Then T has a unique fixed soft set.

Proof Let fix $(a^1,F(a^1))\in(A,F)$ and $(a^1_n,F(a^1_n))=T^n(a^1,F(a^1))$, for $n\in\mathbb{N}$. We present the proof in two steps.

1: We claim

$$\lim_{n\to\infty}\widetilde{G}\Big((a^1_n,F(a^1_n)),(a^1_{n+1},F(a^1_{n+1})),(a^1_{n+1},F(a^1_{n+1}))\Big)=0.$$

Since T is contractive,

$$\begin{aligned}&\widetilde{G}\big((a^1_n,F(a^1_n)),(a^1_{n+1},F(a^1_{n+1})),(a^1_{n+1},F(a^1_{n+1}))\big)\\&=\widetilde{G}\big(T^n(a^1,F(a^1)),T^{n+1}(a^1,F(a^1)),T^{n+1}(a^1,F(a^1))\big)\\&\preceq\alpha\Big[\widetilde{G}\Big((T^{n-1}(a^1,F(a^1))),(T^n(a^1,F(a^1))),(T^n(a^1,F(a^1)))\Big)\Big]\\&\Big[\widetilde{G}\Big((T^{n-1}(a^1,F(a^1))),(T^n(a^1,F(a^1))),(T^n(a^1,F(a^1)))\Big)\Big]\\&=\alpha\Big[\widetilde{G}\Big((a^1_{n-1},F(a^1_{n-1})),(T(a^1_n,F(a^1_n))),(T(a^1_n,F(a^1_n)))\Big)\Big]\\&\Big[\widetilde{G}\Big((T(a^1_{n-1},F(a^1_{n-1}))),(T(a^1_n,F(a^1_n)),(T(a^1_n,F(a^1_n))\Big)\Big]\\&\preceq\Big[\widetilde{G}\Big((a^1_{n-1},F(a^1_{n-1}))),(a^1_n,F(a^1_n)),(a^1_n,F(a^1_n))\Big)\Big]\end{aligned}$$

Thus

$$\widetilde{G}\big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\big) \\ \preceq \widetilde{G}\big((a^1_{n-1}, F(a^1_{n-1})), (a^1_n, F(a^1_n)), (a^1_n, F(a^1_n))\big)$$

so the sequence $\Big\{\widetilde{G}\big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\big)\Big\}$ is decreasing. Then

$$\lim_{n\to\infty} \widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big) = (a^2, r).$$

Assume toward a contradiction that $(a^2, r) \neq 0$ or $r \neq 0$. By (5.7) we have

$$\widetilde{G}\Big((a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+2}, F(a^1_{n+2})), (a^1_{n+2}, F(a^1_{n+2}))\Big) \\ \preceq \alpha\Big[\widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big)\Big] \\ \widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big).$$

Then letting $n \longrightarrow \infty$, we have

$$(a^2, r) \preceq \\ \lim_{n\to\infty} \alpha\Big[\widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big)\Big](a^2, r). \tag{24}$$

Then

$$\frac{1}{r}(a^2, 1) \preceq \\ \frac{1}{r}\Big(\lim_{n\to\infty} \alpha\Big[\widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big)\Big] \\ (a^2, 1)\Big). \tag{25}$$

This follows

$$(a^2, 1) \preceq (a^2, \lim_{n\to\infty} \alpha\Big[\widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big)\Big].$$

Thus,

$$\lim_{n\to\infty} \alpha\Big[\widetilde{G}\Big((a^1_n, F(a^1_n)), (a^1_{n+1}, F(a^1_{n+1})), (a^1_{n+1}, F(a^1_{n+1}))\Big)\Big] \geq 1.$$

This is a contradiction and shows that our claim is true.

2. We claim $\{(a^1_n, F(a^1_n)\}$ is a G-Cauchy soft sequence. By using reductio ad absurdum, let

$$\lim_{m,n\to\infty} \sup \widetilde{G}\big((a^1_n, F(a^1_n)), (a^1_m, F(a^1_m)), (a^1_m, F(a^1_m))\big) \succ (a^1, 0),$$

for every $a^1 \in A$. By the triangle inequality:

$$\begin{aligned}
\widetilde{G}((a_n^1, F(a_n^1)),&(a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))) \\
&\preceq \widetilde{G}((a_n^1, F(a_n^1)), (a_{n+1}^1, F(a_{n+1}^1)), (a_{n+1}^1, F(a_{n+1}^1))) \\
&+ \widetilde{G}((a_{n+1}^1, F(a_{n+1}^1)), (a_{m+1}^1, F(a_{m+1}^1)), (a_{m+1}^1, F(a_{m+1}^1))) \\
&+ \widetilde{G}((a_{m+1}^1, F(a_{m+1}^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))) \\
&\preceq \widetilde{G}((a_n^1, F(a_n^1)), (a_{n+1}^1, F(a_{n+1}^1)), (a_{n+1}^1, F(a_{n+1}^1))) \\
&+ \widetilde{G}((a_{n+1}^1, F(a_{n+1}^1)), (a_{m+1}^1, F(a_{m+1}^1)), (a_{m+1}^1, F(a_{m+1}^1))) \\
&+ \alpha\Big[\widetilde{G}\Big((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \\
&\quad \Big[\widetilde{G}\Big((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big]
\end{aligned}$$

$$\begin{aligned}
\Big[1 - \alpha\Big(&\widetilde{G}((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \\
&\Big[\widetilde{G}\Big((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \\
&\preceq \widetilde{G}((a_n^1, F(a_n^1)), (a_{n+1}^1, F(a_{n+1}^1)), (a_{n+1}^1, F(a_{n+1}^1))) \\
&\quad + \widetilde{G}((a_{m+1}^1, F(a_{m+1}^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))).
\end{aligned}$$

from 1. we have

$$\begin{aligned}
\Big[1 - \lim_{n\to\infty} \alpha\Big(&\widetilde{G}((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \\
&\Big[\widetilde{G}\Big((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \preceq (a^1, 0).
\end{aligned}$$

for every $a^1 \in A$. So, one of the following conditions be happen

(i) $1 - \lim_{n\to\infty} \alpha\Big(\widetilde{G}((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] = 0.$

(ii) $\Big[\widetilde{G}\Big((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] = (a^1, 0)$, for all $a^1 \in A$.

According to the assumption, the case (ii) does not happen, that is

$$\lim_{n\to\infty} \alpha\Big(\widetilde{G}((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] = 1.$$

Definition of α implies that

$$\widetilde{G}((a_n^1, F(a_n^1)), (a_m^1, F(a_m^1)), (a_m^1, F(a_m^1))\Big)\Big] \longrightarrow (a^1, 0)$$

for all $a^1 \in A$.

Let $(a^1, F(a^1)) \in (F, A)$. Since (F, A) is a complete soft G-metric space and $\{T^n(a^1, F(a^1))\}$ is a G-Cauchy sequence, so, there exist a $(x^3, F(x^3)) \in (A, F)$ such that $\lim_{n\to\infty} T^n(a^1, F(a^1)) = (x^3, F(x^3))$ and since T is continuous,

$T(x^3, F(x^3)) = (x^3, F(x^3))$.
For niqueness, suppose that $(a^2, F(a^2))$ another soft fixed point of T, that is $T(a^2, F(a^2)) = (a^2, F(a^2)$ from the contractive condition on T we have:

$$\begin{aligned}\widetilde{G}\Big(T((a^1, F(a^1))),&T((a^2, F(a^2))), T((a^2, F(a^2)))\Big)\\ &= \widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big)\\ &\preceq \alpha\Big(\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big)\Big)\\ &\quad \widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big), \qquad (26)\end{aligned}$$

Thus

$$\begin{aligned}&\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big)\\ &\qquad (1-\alpha\Big(\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big)\Big)) \preceq 0.\end{aligned}$$

Therefore

$$\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big) = 0$$

or

$$(1-\alpha\Big(\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a^2, F(a^2))\Big)\Big)) = 0,$$

From the definition of soft G-metric and property α, it follows that $T(a^2, F(a^2)) = (a^2, F(a^2) = (a^1, F(a^1))$

Example 3 As in Example 1, let $X = [0, \infty]$, $d(x^1, x^2) = |x^1 - x^2|$, $A = \mathbb{Q}^+ = \{x^1 \in \mathbb{Q} : x^1 > 0\}$ and let (F, A) be a soft set on X such that every $(a^1, F(a^1)) \in (F, A)$ defined as $(a^1, F(a^1)) = [0, a^1]$. Consider the mapping $\widetilde{G} : (F, A) \times (F, A) \times (F, A) \longrightarrow (A, \mathbb{R}^+ \cup \{0\})$ defined by

$$\begin{aligned}\widetilde{G}((a^1, F(a^1)),&(a^2, F(a^2)), (a'^3, F(a'^3)))\\ &= \max\{|a^1 - a^2|, |a^2 - a'^3|, |a'^3 - a^1|).\end{aligned}$$

Let $T : ((F, A), \widetilde{G}) \longrightarrow ((F, A), \widetilde{G})$ be a soft function defined by

$$T((a^1, F(a^1))) = (\frac{1}{4}a^1, F(\frac{1}{4}a^1)) = [0, \frac{1}{4}a^1].$$

and

$$\begin{aligned}\widetilde{G}\Big(T((a^1,&F(a^1))), T((a^2, F(a^2))), T((a'^3, F(a'^3)))\Big)\\ &= \widetilde{G}\Big((\frac{1}{4}a^1, F(\frac{1}{4}a^1)), (\frac{1}{4}a^2, F(\frac{1}{4}a^2)), (\frac{1}{4}a'^3, F(\frac{1}{4}a'^3))\Big)\\ &= max\{|\frac{1}{4}a^1 - \frac{1}{4}a^2|, |\frac{1}{4}a^2 - \frac{1}{4}a'^3|, |\frac{1}{4}a'^3 - \frac{1}{4}a^1|)\\ &\preceq \frac{1}{4}max\{|a^1 - a^2|, |a^2 - a'^3|, |a'^3 - a^1|)\\ &= \frac{1}{4}\widetilde{G}\Big((a^1, F(a^1)), (a^2, F(a^2)), (a'^3, F(a'^3))\Big)\end{aligned}$$

Now define $\alpha : (A, \mathbb{R}^+) \to [0, 1)$ with

$$\alpha(a^1, x^1) = \begin{cases} 1 - \frac{1}{16x+4} & x^1 \geq 1 \\ 1 - \frac{1}{20}x^1 & 0 \leq x^1 < 1. \end{cases}$$

Clearly, $\alpha(a^1, x^1) \geq \frac{1}{4}$, for every $x^1 > 0$ and $a^1 \in A$. Now, by using Theorem 10, It results in that T has a unique fixed soft set.

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