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Deterministic Model of Corruption Dynamics in Nigeria VIA Homotopy Perturbation Method

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Abstract Corruption is a slowly decaying poison in Nigeria. Corruption is a global problem that individuals in a community can be exposed to. This paper developed the dynamics of corruption and the compartments were divided into six sections: Susceptible, Exposed, Corrupt, Honest, Punished and Recovered. The paper was designed to deal with the stability of corrupt individuals and, using the homotopy perturbation technique, the model equations are solved for simulations to performed numerically. The analysis findings demonstrate that the corruption-free equilibrium is locally asymptotically stable if $R_0 < 1$, indicating that there is corruption in the population. disappears and if $R_0 > 1$, means that the number of corruption rises per-capital in a society. Also from the results, the homotopy perturbation method shows accuracy and convergence very quickly for numerical simulations despite it require perturbation for convergent. The observations and suggestions are outlined to have a corruption-free society.

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1 Introduction

The term corruption according to [12] and [23] corruption is an unlawful practice done for personal benefit in a variety of sectors, including law enforcement, security services, public service and oil and the electoral system. Olujobi, (2023) in [20] claims that the failure of anti-corruption organizations in Nigeria can be attributed to a combination of factors, including inadequate finance, a lack of political will on the part of the government to combat corruption, and a poor use of information and communication technology (ICT). The money laundering prohibition, the Economic and Financial Crimes Commission (EFCC) Act of 2004, and the Independent Corrupt Practices and Other Related Offences Commission (ICPC) Act of 2000 Act 2011 as well as the Tribunal and Code Act conduct of 1991 are just a few of the laws and organizations that Nigeria has adopted to fight corruption. These laws and institutions aim to prevent, investigate, prosecute and punish corruption and related crimes, as well as recover stolen property and promote transparency and accountability in public affairs. Corruption remains pervasive and persistent in Nigeria despite these legal and institutional frameworks. Also, according to Olujobi, (2023) in [20] the Nigerian Corruption Index as of September 2020 indexed 149 out of 180 countries with a score of 25 percent out of 100 percent, indicating high levels of perceived corruption in the public sector. The report also highlighted some of the issues hindering the effectiveness of anti-corruption efforts in Nigeria, such as political interference, lack of independence and capacity of anti-corruption agencies, a weak judicial system, poor enforcement of laws and sanctions, low public awareness and participation, and inadequate protection for whistle-blowers and activists. The consequences of corruption in Nigeria have been found to affect not only government and the economy but also the social fabric of society, promoting inequality, undermining democracy and exacerbating poverty and insecurity in [20].

Corruption in Nigeria has significantly affected the provision of basic services to citizens, especially in the areas of health and education. Corruption often leads to miscalculation and misappropriation of public funds intended to improve these critical areas. This leads to poor service delivery and lack of access to necessities for citizens in [20]. However, there is still no significant reduction in quantity; even now it is starting to reach villages with village funds paid by the government of Nigeria. Therefore, in [19], they studied and examined the effectiveness of anti-corruption laws in Nigeria through a comparative analysis of legal frameworks and practical outcomes. The findings suggest that despite strong legal frameworks to fight corruption, operational outcomes may have been more effective due to several factors. In [9] examine the mechanics of corruption as well as the three preventative strategies suggested to deal with corruption in the Nigerian system. The dynamics of the corruption model were described. The threshold for removing corruption is set by the basic reproduction number. Pontryagin's maximal principle was applied in the optimal control technique to assess the effectiveness of the recommended control measures. Similarly, Umar,*et al.,* (2021) in [24] developed a model with effective tools used in Indonesia to detect corruption measures. The results of the study, shows that the tools reduced complicated corruption in a government sectors. Also in [4], modelling and analysis of corruption dynamics involving media coverage has been built in to address this threat. A deterministic mathematical model of the Zika virus was constructed by Adamu, *et al.,* (2017) in [1], using two control strategies: human therapy and pesticide spray for mosquitoes. The model's approximate solution was obtained through the application of the homotopy perturbation approach. [6] built a mathematical model of the dynamics of corruption and included approaches for optimal control to measure the behaviour of individuals in a population.[9] analysed tactics for preventing and controlling corruption dynamics. The reintegration of those who have recovered from corrupt practices back into society is the main emphasis of the study.

Furthermore, in [2] developed a corruption model with optimal control strategies. The dynamics of corruption were analysed and a non-linear deterministic model was suggested. In [16] developed a model that describes prevention and disengagement strategies. Similarly, [17] developed a model that mathematically describes strategies to prevent corruption and disengagement using an epidemiological compartmental model. Furthermore, [21] developed a model of corruption dynamics of a nonlinear system of differential equations with four sections viz. susceptible, exposed, infected and punished. Similarly, [5] developed and examined a compartmentalized mathematical model of the dynamics of corruption and subdivide into five. A model of corruption in Nigeria was developed and examined by [7]. Also, A mathematical model of corruption that takes into account the awareness raised by anti-corruption and jail counselling was proposed by [13]. A novel mathematical model for the dynamics of moral corruption with age-appropriate sexual information and guidance and counselling was developed by Mokaya *et al.,* (2021) in [14]. Nwajeri *et al.,* (2023) developed a mathematical model of corruption dynamics that includes a fractal-fractional derivative in [18]. The model was intended to study whether corruption exists among adults in government, adults in civil services and public authority, youth in primary schools, youth in tertiary institutions, and individuals who have given up engaging in corrupt practices.

This paper therefore aims to extend the existing model in [2] by incorporating the fair group method, the group penalty method and the homotopy perturbation method for semi-analytic solutions.

2 Material and Methods

The homotopy perturbation approach is used to solve the model equations, which are broken down into six (6) compartments for analysis. The model diagram can be deduced in Fig. 1. The model equations are given by the corresponding diagram in Fig. 1 as

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda - \rho\beta SC + (1-\theta)\,\varepsilon R - (\mu+\kappa)\,S, \\ \frac{dE(t)}{dt} = \rho\beta SC - (\mu+\delta)\,E, \\ \frac{dC(t)}{dt} = \alpha\delta E - (\mu+\tau+\sigma)\,C, \\ \frac{dP(t)}{dt} = \sigma C - (\mu+\pi)\,P, \\ \frac{dH(t)}{dt} = \kappa S + \theta\varepsilon R - \mu H, \\ \frac{dR(t)}{dt} = (1-\alpha)\,\delta E + \tau C + \pi P - (\mu+\theta\varepsilon)\,R, \end{cases} \tag{1}$$

with the initial conditions as

$$S(0) = S_0 > 0, \qquad E(0) = E_0 \geq 0, \qquad C(0) = C_0 \geq 0,$$
$$P(0) = P_0 \geq 0, \qquad H(0) = H_0 \geq 0, \qquad R(0) = R_0 \geq 0.$$

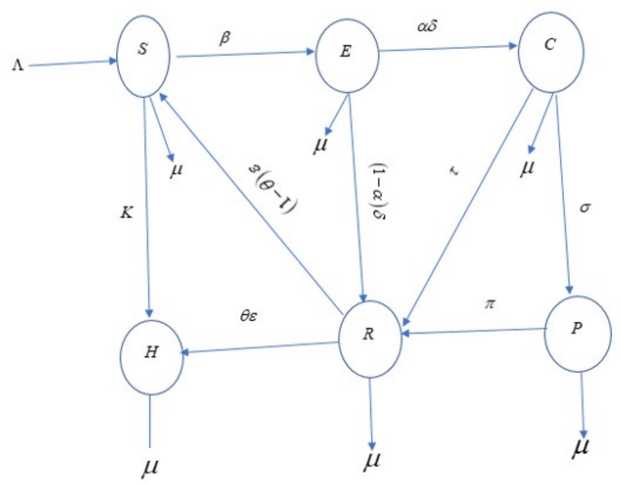

Fig. 1 Corruption diagram of the model

Table 1 Description of Variables and Parameters

Symbol	**Description**
S	Susceptible humans
E	Exposed Individuals
C	Corruption
P	Punished
H	Honest
R	Recovered
Λ	Recruitment rate of population
ρ	The transmission rate per contact
β	Contact of individuals rate per contact
δ	The rate at which those who are exposed get corrupted
σ	The rate at which corrupted individuals moved to punish group
ϵ	The rate at which individuals who have recovered become honest
τ	Rate at which corrupted individuals become punished
π	The rate of punished individuals
κ	proportion rate of individuals that join the honest population
μ	Natural mortality rate for all individuals
α	The rate of individuals who leave the exposed group and enter the corrupted subpopulation
θ	The proportion rate of individuals who leave the recovered group and enter the honest subgroup

2.1 The Model Formation

The population $N(t)$ is subdivided into six groups. Those who are susceptible to corruption $S(t)$, those who exposed to a corrupt are $E(t)$, those involved in corrupt practitioners, are corrupt individuals $C(t)$, those who have stopped engaging in corrupt practices are healed individuals $R(t)$, those caught in corrupt practices are punished individuals $P(t)$ and those who abstain from corruption and refuse to engage in the act are honest individuals $H(t)$ at time $t \geq 0$.
The Assumptions: The assumptions of the model are given by recruitment rate Λ to a receptive class, like immigration or birth.This also assume that κ joins the honest subpopulation that never partakes in corruption activities. Taking into account natural death rate μ is for all individuals throughout the entire study period. With a chance of ρ every interaction, susceptible people will come into touch with corrupted individuals at level β and transition to the exposed class. Among them, α moved at a pace of δ to the damaged partition, whereas the remaining individuals moved to the restored partitions. Corrupted individuals learn about the prison corruption effect and move to the restored subpopulation in proportion σ. Of these cured individuals, θ moved at ε to the sensitive compartment and the other part joined the honesty compartment. These individuals, τ, moved to the punished class, and finally π individuals moved to the restored class.

2.2 Positive Invariant Region

The rate of change of the total population of corruption given as follows:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dC}{dt} + \frac{dP}{dt} + \frac{dH}{dt} + \frac{dR}{dt}, \tag{2}$$

$$\frac{dN}{dt} = \Lambda - \mu N. \tag{3}$$

Theorem 1 *The region* $\eta = (S, E, C, P, H, R) \in \Re^6_+$ *is positively invariant and attracts all solutions of the system equations.*

Proof Let Assume there is no corruption induced death in the population equation, which gives

$$\frac{dN}{dt} \leq \Lambda - \mu N, \tag{4}$$

$$\frac{dN}{dt} + \mu N \leq \Lambda. \tag{5}$$

Multiplying (5) with its integrating factor, gives

$$\frac{dN}{dt} \exp \mu t + \mu N e^{\mu t} \leq \Lambda \exp \mu t, \tag{6}$$

$$\frac{d}{dt}\left[N e^{\mu t}\right] \leq \Lambda \exp \mu t. \tag{7}$$

Integrating equation (7), gives

$$N \exp \mu t \leq \frac{\Lambda \exp \mu t}{\mu} + A. \tag{8}$$

Divide (8) by $\exp \mu t$ gives

$$N(t) \leq \frac{\Lambda}{\mu} + A \exp -\mu t. \tag{9}$$

Thus, $t = 0$

$$N(0) - \frac{\Lambda}{\mu} \leq N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right) \exp -\mu t. \tag{10}$$

Taking the limit as $t \to \infty$,

$$N \leq \frac{\Lambda}{\mu}. \tag{11}$$

As $t \to \infty$, the inequalities (11) shows that the total population of (N) approaches to $\frac{\Lambda}{\mu}$, where $\frac{\Lambda}{\mu}$ are upper bounds. This means that at any time t, all solutions with initial conditions in η remain in η. Hence, the solutions of the model at each time t enter the feasible region η. Therefore, the solution of system (1) is positively invariant in the region η, which is mathematically well located.

2.3 Positivity of Solutions for the Model

This paper focuses on corruption populations and it is necessary to show all the states variables are positive at time t.

Theorem 2 *Let the initial conditions be $S > 0$, $E > 0$, $C > 0$, $P > 0$, $H > 0$, and $R > 0$. The solution of (S, E, C, P, H, R) of the model system (1-6) is positive forall $t > 0$.*

Proof Suppose there is no corruption induced death in the population equation, which gives; $S+E+C+P+H+R = N$. For all t > 0,$S \le N$, $E \le N$, $C \le N$, $P \le N$, $H \le N$, and $R \le N$. From the system of equations (1) gives

$$\frac{dS(t)}{dt} = \Lambda - \rho\beta SC + (1-\theta)\varepsilon R - (\mu+\kappa)S \ge -(\mu+\kappa)S. \tag{12}$$

Thus, by reduction the (12) gives

$$\frac{dS(t)}{dt} \ge -(\mu+\kappa)S, \tag{13}$$

$$\int \frac{dS(t)}{S} \ge \int -(\mu+\kappa)\,dt, \tag{14}$$

$$\ln\, S \ge -(\mu+\kappa)t + c, \tag{15}$$

$$S \ge \exp\{-(\mu+\kappa)t + c\}, \tag{16}$$

$$S \ge \kappa \exp\{-(\mu+\kappa)t\}. \tag{17}$$

Therefore, $S(t) \ge S(0)\exp\{-(\mu+\kappa)t\} \ge 0$, which applied to all equations in model system (1) and obtained;

$$\begin{cases} E(t) \ge E(0)\exp -(\mu+\delta)t \ge 0, \\ C(t) \ge C(0)\exp -(\mu+\tau+\sigma)t \ge 0, \\ P(t) \ge P(0)\exp -(\mu+\pi)t \ge 0, \\ H(t) \ge H(0)\exp -\mu t \ge 0, \\ R(t) \ge R(0)\exp -(\mu+\varepsilon)t \ge 0. \end{cases} \tag{18}$$

Thus, this shows that the solutions of the model are exist and unique, which means the corruption can be control in a population. This complete the proved.

2.4 The Basic Reproduction Number R_0

Is characterized as a novel infectious potential spread by an individual within a group. in [8]. The basic reproductive number R_0 is defined as the average number of secondary cases produced by a corrupt individual in an otherwise susceptible host population in [22]. The basic reproduction number R_0 can also mean the number of individuals corrupted during their entire corruption period in a population that is completely susceptible in [25]. The basic reproduction number R_0 has no unit. When $R_0 < 1$, the corruption dies out in the population. Otherwise, when $R_0 > 1$, the corruption persist.The fundamental reproduction number in this model is obtained using the next generation matrix method R_0, as stated in [25]. The incidence rate of new corruption in partition i is represented by $f_i(x)$, and the basic reproduction number $(R_0) = \rho\left(FV^{-1}\right)$. V_i^+ the rate of movement of individuals into compartment i by all other means, and V_i^- the rate of movement of individuals out of compartment i . In model equation(1), the infected compartments include $E(t), C(t), P(t)$ and the expected secondary corruption depends on these classes. The rate of occurrence of new corruption in compartment i is given by the matrix.

$$F = \begin{pmatrix} \rho\beta SC \\ 0 \\ \sigma C \end{pmatrix}. \tag{19}$$

At the corruption-free equilibrium point, the jacobian matrix of F is evaluated as follows:

$$F = \left(\frac{\partial f_i\left(E^0\right)}{\partial x_j}\right),$$

where $x_j = E, C, P$ forall $j = 1, 2, 3$ and E^0 is the corruption-free equilibrium. At the corruption-free equilibrium point, the Jacobian matrix of (19) is evaluated yield

$$F = \begin{pmatrix} 0 & \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu} & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & 0 \end{pmatrix}, \qquad V^{-1} = \begin{pmatrix} \frac{1}{(\delta+\mu)} & 0 & 0 \\ \frac{-\alpha\delta}{(\tau+\sigma+\mu)} & \frac{1}{(\tau+\sigma+\mu)} & 0 \\ 0 & 0 & \frac{1}{(\pi+\mu)} \end{pmatrix}. \tag{20}$$

The next generated matrix FV^{-1} is given as

$$FV^{-1} = \begin{pmatrix} 0 & \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu} & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(\delta+\mu)} & 0 & 0 \\ \frac{-\alpha\delta}{(\tau+\sigma+\mu)} & \frac{1}{(\tau+\sigma+\mu)} & 0 \\ 0 & 0 & \frac{1}{(\pi+\mu)} \end{pmatrix}. \tag{21}$$

Therefore, multiply 21 gives

$$\begin{pmatrix} \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu(\delta+\mu)} & \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{\sigma}{(\tau+\sigma+\mu)} & 0 \end{pmatrix}. \tag{22}$$

Then $\rho\left(FV^{-1}\right)$ is the dominant eigenvalue of the $\left(FV^{-1}\right)$ matrix. Obtained

$$A = \begin{pmatrix} \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu(\delta+\mu)} & \frac{\rho\beta\Lambda k}{(\mu+\kappa)\mu} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{\sigma}{(\tau+\sigma+\mu)} & 0 \end{pmatrix},$$

and

$$|A-\lambda I| = \begin{pmatrix} \frac{\rho\beta k}{(\mu+\kappa)\mu(\delta+\mu)} & \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{\sigma}{(\tau+\sigma+\mu)} & 0 \end{pmatrix} - \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

where I is the identity matrix and λ is the arbitrary sing within the diagonal of the identity matrix; which obtained as

$$\begin{pmatrix} \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu(\delta+\mu)} - \lambda_1 & \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\mu} & 0 \\ 0 & -\lambda_2 & 0 \\ 0 & \frac{\sigma}{(\tau+\sigma+\mu)} & -\lambda_3 \end{pmatrix} = 0.$$

The determinant is the eigenvalue and the root of the characteristic polynomial is the basic reproduction number, which is given as

$$R_0 = \frac{\rho\beta\Lambda\kappa}{(\mu+\kappa)\,\mu\,(\delta+\mu)}. \tag{23}$$

2.5 Corruption-Free Equilibrium Point (CFEP)

Let $E_0 = (S, E, C, P.H, R) = (S_0, E_0, C_0, P_0.H_0, R_0)$ as given in (1). Therefore, the corruption free equilibrium state (CFEP) given as

$$E_0 = \left(\frac{\Lambda k}{(\mu+\kappa)\,\mu}, 0, 0, 0, 0, \frac{\Lambda}{(\mu+\kappa)} \right). \tag{24}$$

2.6 Local Stability of Corruption-Free Equilibrium Point (LSCFEP)

Using the CFEP, the Jacobian matrix of system (1) is analysed in order to examine the local stability at the corruption-free equilibrium point. Next, the sign of the Jacobian matrix's eigenvalues is used to calculate the stability.

Theorem 3 *The local stability of corruption-free equilibrium is said to be locally asymptotically stable, if all the eigenvalues of the Jacobian matrix are negative.*

Proof Supposing the Jacobian matrix of the model equations is given as

$$J = \begin{bmatrix} -\rho\beta C - (\mu+\kappa) & 0 & -\rho\beta S & 0 & 0 & (1-\theta)\varepsilon \\ \rho\beta C & -(\mu+\delta) & \rho\beta S & 0 & 0 & 0 \\ 0 & \alpha\delta & -(\tau+\mu+\delta) & 0 & 0 & 0 \\ 0 & 0 & \sigma & -(\mu+\pi) & 0 & 0 \\ \kappa & 0 & 0 & 0 & -\mu & \theta\varepsilon \\ 0 & (1-\alpha)\delta & \tau & \pi & 0 & -(\theta\varepsilon+\mu) \end{bmatrix}.$$

At the corruption-free equilibrium point, evaluating system equation (1) yields

$$J(E_0)=\begin{bmatrix} -(\mu+\kappa) & 0 & -\frac{\rho\beta\Lambda k}{(\mu+\kappa)\mu} & 0 & 0 & (1-\theta)\varepsilon \\ 0 & -(\mu+\delta) & \frac{\rho\beta\Lambda k}{(\mu+\kappa)\mu} & 0 & 0 & 0 \\ 0 & \alpha\delta & -(\tau+\mu+\delta) & 0 & 0 & 0 \\ 0 & 0 & \sigma & -(\mu+\pi) & 0 & 0 \\ \kappa & 0 & 0 & 0 & -\mu & \theta\varepsilon \\ 0 & (1-\alpha)\delta & \tau & \pi & 0 & -(\theta\varepsilon+\mu) \end{bmatrix}.$$

The characteristic polynomial derived by solving the Jacobian matrix of the system (1) at corruption-free equilibrium obtained

$$((\mu+\lambda)(\mu+\theta\varepsilon+\lambda)(\mu+k+\lambda)(\mu+\pi+\lambda))[\lambda^2+m_1\lambda+m_2]=0. \tag{25}$$

where

$$m_1 = 2k+\tau+\delta+\sigma,$$

$$m_2 = \frac{-\alpha\delta\rho\beta\Lambda\kappa+(\mu+\kappa)\mu(\mu+\delta)(\sigma+\mu)}{(\mu+\kappa)\mu},$$

and λ is the coefficient values from the lift hand side in (25), which gives

$$\begin{aligned} \lambda_1 &= -\mu < 0, \\ \lambda_2 &= -(\mu+\pi) < 0, \\ \lambda_3 &= -(\mu+k) < 0, \\ \lambda_4 &= -(\mu+\theta\varepsilon) < 0. \end{aligned}$$

From the last expression in (25), the right hand side is resolved to quadratic equation, which is given as

$$\lambda^2+m_1\lambda+m_2=0. \tag{26}$$

This result shows that the corruption-free equilibrium is locally asymptotically stable if and only if $R_0 < 1$, or otherwise unstable when $R_0 > 1$. According to the Routh-Hurwitz criteria in [15], the criterion was applied to (1) and has a strictly negative real root when $m_1 > 0, m_2 > 0$. From this it can be clearly deduced that $m_1 > 0$, which is positive and moreover given as

$$\begin{aligned} m_2 &= \frac{-\alpha\delta\rho\beta\Lambda k+(\mu+k)\mu(\mu+\delta)(\sigma+\mu)}{(\mu+k)\mu} \\ &= (\mu+\delta)(\sigma+\mu)(1-R_0) > 0. \end{aligned} \tag{27}$$

The Ruth-Hurwitz criterion states that all roots of a polynomial of determinants must have negative real parts iff

$$b_1>0, \quad b_2>0, \quad b_3>0, \quad b_4>0, \quad b_5>0, \quad b_6>0,$$

$$b_1b_2b_3 > b_3^2+b_1^2b_4, \quad (b_1b_4-b_5)(b_1b_2b_3-b_3^2+b_1^2b_4) > b_5(b_1b_2-b_3)^2+b_1b_5^2.$$

Subsequently, for endemic of corruption if $R_0 > 1$, E^* is locally asymptotically stable if satisfied the above assertion and endemic equilibrium exists.
Finally, the corruption free-equilibrium point is therefore locally asymptotically stable iff $R_0 < 1$. Hence, the proved.

2.7 Global Stability of Corruption Free Equilibrium

Theorem 4 *The stability of corruption free-equilibrium point E_0 of the system (1) is globally asymptotically stable iff $R_0 < 1$.*

Proof following the Lyapunov function, Supposing

$$L = a_1 E + a_2 C. \tag{28}$$

Differentiate (28) and substituting the values of E and C in (28) gives

$$\begin{aligned}\frac{dL}{dt} &= a_1 \left[\rho\beta SC - (\delta+\mu)E\right] + a_2 \left[\alpha\delta E - (\sigma+\mu+\tau)C\right] \\ &= a_1\rho\beta SC - a_2(\sigma+\mu+\tau)C - a_1(\delta+\mu)E + a_2\alpha\delta E.\end{aligned} \tag{29}$$

Thus, let $a_1 = \left(\frac{\alpha\delta}{\delta+\mu}\right) a_2$ and also substituting the value of $a_1$in (29) obtained

$$\begin{aligned}\tfrac{dL}{dt} &= \tfrac{\alpha\delta}{\delta+\mu} a_2\rho\beta SC - a_2(\sigma+\mu+\tau)C, \\ &\leq \tfrac{\alpha\delta\rho\beta\Lambda\kappa}{(\delta+\mu)(\mu+\kappa)\mu} - (\sigma+\mu+\tau)a_2 C.\end{aligned} \tag{30}$$

Putting $a_1 = 1$ and substituting R_0, and obtained

$$\frac{dL}{dt} \leq (\sigma+\mu+\tau)(R_0 - 1)C. \tag{31}$$

Therefore, $\frac{dL}{dt} \leq 0$ for $C \leq 0$ this show that $R_0 < 1$ and $\frac{dL}{dt} = 0$ iff $C = 0$. This implies that $\frac{dL}{dt} \leq 0$ which is E_0. According to LaSalle's invariance principle E_0 is globally asymptotically stable in η.

2.8 Analytical solution using Homotopy Perturbation Method (HPM)

Ji-Haun (2000) originally put forward the fundamentals of the Homotopy perturbation approach in [10]. Many linear and non-linear equations can be solved analytically approximatively with the Homotopy perturbation method. In [11], A series expansion method for solving non-linear partial differential equations is the homotopy perturbation method. The following non-linear differential equation was considered in [11] to demonstrate the fundamental concepts of this methodology.
The following equations are provided as follows

$$A_3\left(U\right) - f\left(r\right) = 0, \quad r \in \varOmega. \tag{32}$$

Subject to the boundary condition

$$B_3\left(U, \frac{\partial U}{\partial n}\right) = 0, \quad r \in \varGamma, \tag{33}$$

where A_3 is a general differential operator, B_3 a boundary operator, $f\left(r\right)$ is a known analytical function and $\varGamma$ is the boundary of the domain $\varOmega$. The operator A_3 can be divided into two parts L and N, where L is the linear part, and N is the non-linear part. The Homotopy perturbation structure is shown as follows

$$H(V,h) = (1-h)[L(V) - L(U_0)] + h[A(V) - f(r)] = 0, \tag{34}$$

where $V(r,P) : \Omega \in [0,1] \to R$. $P \in [0,1]$ is the parameter for embedding, and U_0 is an approximation that meets the boundary requirement. It is likely that a power series in h can be used to express (34) as

$$V = V_0 + hV_1 + h^2V_2 + \cdots \tag{35}$$

The following gives

$$U = \lim_{h\to 1} v = v_0 + hv_1 + h^2v_2 + \cdots \tag{36}$$

In most circumstances, the series (36) is convergent. Nonetheless, the non-linear operator determines the convergent rate.

2.9 Solution of the Model Equations

The system equations (1) have the following differential equations

$$\left.\begin{aligned}
&\tfrac{dS}{dt} + \rho\beta SC + (\mu+\kappa)S - (1-\theta)\varepsilon R - \Lambda,\\
&\tfrac{dE}{dt} + (\mu+\delta)E - \rho\beta SC,\\
&\tfrac{dC}{dt} + (\mu+\tau+\sigma)C - \alpha\delta,\\
&\tfrac{dP}{dt} + (\mu+\pi)P - \sigma,\\
&\tfrac{dH}{dt} + \mu H - \kappa S - \theta\varepsilon,\\
&\tfrac{dR}{dt} + (\mu+\varepsilon)R - (1-\alpha)\delta E - \tau C - \pi.
\end{aligned}\right\} = 0 \tag{37}$$

with the following initial conditions

$$\begin{aligned}
&S(0) = S_0, \qquad && E(0) = E_0, \qquad && C(0) = C_0,\\
&P(0) = P_0, && H(0) = H_0, && R(0) = R_0.
\end{aligned}$$

Let

$$S = a_0 + ha_1 + h^2a_2 + \cdots \tag{38}$$

$$E = b_0 + hb_1 + h^2b_2 + \cdots \tag{39}$$

$$C = c_0 + hc_1 + h^2c_2 + \cdots \tag{40}$$

$$P = e_0 + he_1 + h^2e_2 + \cdots \tag{41}$$

$$H = f_0 + hf_1 + h^2f_2 + \cdots \tag{42}$$

$$R = w_0 + hw_1 + h^2 w_2 + \cdots \tag{43}$$

Apply the application of Homotopy perturbation method (HPM) on (37) starting from the first model equations, which follows as

$$(1-h)\frac{dS}{dt} + h\left[\frac{dS}{dt} + \rho\beta SC + (\mu+\kappa)S - (1-\theta)\varepsilon R - \Lambda\right] = 0, \tag{44}$$

$$(1-h)\left(a_0' + ha_1' + h^2 a_2' + \cdots\right) + h\Big[(a_0' + ha_1' + h^2 a_2' + \cdots)$$
$$+ \rho\beta(a_0 + ha_1 + h^2 a_2 + \cdots)(c_0 + hc_1 + h^2 c_2 + \cdots)$$
$$+ (\mu+\kappa)\left(a_0 + ha_1 + h^2 a_2 + \cdots\right)$$
$$- (1-\theta)\varepsilon\left(w_0 + hw_1 + h^2 w_2 + \cdots\right) - \Lambda\Big] = 0,$$

$$(1-h)\left(a_0' + ha_1' + h^2 a_2' + \cdots\right) + h\Big[\left(a_0' + ha_1' + h^2 a_2' + \cdots\right)$$
$$+ \rho\beta\left(a_0 + ha_1 + h^2 a_2 + \cdots\right)\left(c_0 + hc_1 + h^2 c_2 + \cdots\right)$$
$$+ (\mu+\kappa)\left(a_0 + ha_1 + h^2 a_2 + \cdots\right)$$
$$- (1-\theta)\varepsilon\left(w_0 + hw_1 + h^2 w_2 + \cdots\right) - \Lambda\Big] = 0. \tag{45}$$

Substitute equation (38), (39), and (40) into equation (44)

$$\left(a_0' + ha_1' + h^2 a_2' + \cdots\right) + h\Big[\rho\beta\left(a_0 + ha_1 + h^2 a_2 + \cdots\right)\left(c_0 + hc_1 + h^2 c_2 + \cdots\right)$$
$$+ (\mu+\kappa)\left(a_0 + ha_1 + h^2 a_2 + \cdots\right)$$
$$- (1-\theta)\varepsilon\left(w_0 + hw_1 + h^2 w_2 + \cdots\right) - \Lambda\Big] = 0. \tag{46}$$

Coefficients of the powers of h are expanded and obtained

$$h^0 : a_0' = 0, \tag{47}$$
$$h^1 : a_1' + \rho\beta a_0 c_0 + (\mu+\kappa)a_0 - (1-\theta)\varepsilon w_0 - \Lambda = 0, \tag{48}$$
$$h^2 : a_2' + \rho\beta(a_1 c_0 + a_0 c_1) + (\mu+\kappa)a_1 - (1-\theta)\varepsilon w_1. \tag{49}$$

Subsequently, it follows on each model equations in (37) gives

$$(1-h)\frac{dE}{dt} + h\left[\frac{dE}{dt} + (\mu+\delta)E - \rho\beta SC\right] = 0. \tag{50}$$

Substitute (38), (39), and (40) into (49)

$$(1-h)\left(b_0' + hb_1' + h^2 b_2' + \cdots\right) + h\Big[\left(b_0' + hb_1' + h^2 b_2' + \cdots\right)$$

$$+\left(\mu+\delta\right)\left(b_0+hb_1+h^2b_2+\cdots\right)$$

$$-\rho\beta\left(a_0+ha_1+h^2a_2+\cdots\right)\left(c_0+hc_1+h^2c_2+\cdots\right)\Bigg]=0. \quad (51)$$

$$\left(b_0^{'}+hb_1^{'}+h^2b_2^{'}+\cdots\right)+h\Bigg[\left(\mu+\delta\right)\left(b_0+hb_1+h^2b_2+\cdots\right)$$

$$-\rho\beta\left(a_0+ha_1+h^2a_2+\cdots\right)\left(c_0+hc_1+h^2c_2+\cdots\right)\Bigg]=0. \quad (52)$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : b_0^{'}=0, \quad (53)$$

$$h^1 : b_1^{'}+\left(\mu+\delta\right)b_0-\rho\beta a_0c_0=0, \quad (54)$$

$$h^2 : b_2^{'}+\left(\mu+\delta\right)b_1-\rho\beta\left(a_0c_1+a_1c_0\right)=0. \quad (55)$$

Also, apply HPM on the other model equations in (37) gives

$$(1-h)\frac{dC}{dt}+h\left[\frac{dC}{dt}+\left(\mu+\tau+\sigma\right)C-\alpha\delta E\right]=0. \quad (56)$$

Substitute (39) and (40) into (55)

$$(1-h)\left(c_0^{'}+hc_1^{'}+h^2c_2^{'}+\cdots\right)+h\Bigg[\left(c_0^{'}+hc_1^{'}+h^2c_2^{'}+\cdots\right)$$

$$+\left(\mu+\tau+\sigma\right)\left(c_0+hc_1+h^2c_2+\cdots\right)-\alpha\delta\left(b_0+hb_1+h^2b_2+\cdots\right)\Bigg]=0, \quad (57)$$

$$\left(c_0^{'}+hc_1^{'}+h^2c_2^{'}+\cdots\right)+h\Bigg[\left(\mu+\tau+\sigma\right)\left(c_0+hc_1+h^2c_2+\cdots\right)$$

$$-\alpha\delta\left(b_0+hb_1+h^2b_2+\cdots\right)\Bigg]=0, \quad (58)$$

$$(1-h)\left(b_0^{'}+hb_1^{'}+h^2b_2^{'}+\cdots\right)+h\Bigg[\left(b_0^{'}+hb_1^{'}+h^2b_2^{'}+\cdots\right)$$

$$+\left(\mu+\delta\right)\left(b_0+hb_1+h^2b_2+\cdots\right)$$

$$-\rho\beta\left(a_0+ha_1+h^2a_2+\cdots\right)\left(c_0+hc_1+h^2c_2+\cdots\right)\Bigg]=0, \quad (59)$$

$$\left(b_0^{'}+hb_1^{'}+h^2b_2^{'}+\cdots\right)+h\Bigg[\left(\mu+\delta\right)\left(b_0+hb_1+h^2b_2+\cdots\right)$$

$$-\rho\beta\left(a_0+ha_1+h^2a_2+\cdots\right)\left(c_0+hc_1+h^2c_2+\cdots\right)\Bigg]=0. \quad (60)$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : b_0^{'} = 0, \tag{61}$$

$$h^1 : b_1^{'} + (\mu + \delta)\, b_0 - \rho\beta a_0 c_0 = 0, \tag{62}$$

$$h^2 : b_2^{'} + (\mu + \delta)\, b_1 - \rho\beta\, (a_0 c_1 + a_1 c_0) = 0. \tag{63}$$

Then applying HPM to (37)

$$(1-h)\,\frac{dC}{dt} + h\left[\frac{dC}{dt} + (\mu + \tau + \sigma)\, C - \alpha\delta E\right] = 0. \tag{64}$$

Substitute (38) and (39) into (63)

$$(1-h)\left(c_0^{'} + hc_1^{'} + h^2 c_2^{'} + \cdots\right) + h\Bigg[\left(c_0^{'} + hc_1^{'} + h^2 c_2^{'} + \cdots\right)$$
$$+ (\mu + \tau + \sigma)\left(c_0 + hc_1 + h^2 c_2 + \cdots\right) - \alpha\delta\left(b_0 + hb_1 + h^2 b_2 + \cdots\right)\Bigg] = 0, \tag{65}$$

$$\left(c_0^{'} + hc_1^{'} + h^2 c_2^{'} + \cdots\right) + h\Bigg[(\mu + \tau + \sigma)\left(c_0 + hc_1 + h^2 c_2 + \cdots\right)$$
$$- \alpha\delta\left(b_0 + hb_1 + h^2 b_2 + \cdots\right)\Bigg] = 0. \tag{66}$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : c_0^{'} = 0, \tag{67}$$

$$h^1 : c_1^{'} + (\mu + \tau + \sigma)\, c_0 - \alpha\delta b_0 = 0, \tag{68}$$

$$h^2 : c_2^{'} + (\mu + \tau + \sigma)\, c_1 - \alpha\delta b_1 = 0. \tag{69}$$

Applying HPM to (37) also gives

$$(1-h)\,\frac{dP}{dt} + h\left[\frac{dP}{dt} + (\mu + \pi)\, P - \sigma C\right] = 0. \tag{70}$$

Substitute (40) and (41) into (69)

$$(1-h)\left(e_0^{'} + he_1^{'} + h^2 e_2^{'} + \cdots\right) + h\Bigg[\left(e_0^{'} + he_1^{'} + h^2 e_2^{'} + \cdots\right)$$
$$+ (\mu + \pi)\left(e_0^{'} + he_1^{'} + h^2 e_2^{'} + \cdots\right) - \sigma\left(c_0^{'} + hc_1^{'} + h^2 c_2^{'} + \cdots\right)\Bigg] = 0, \tag{71}$$

$$\left(e_0^{'} + he_1^{'} + h^2 e_2^{'} + \cdots\right) + h\Bigg[(\mu + \pi)\left(e_0^{'} + he_1^{'} + h^2 e_2^{'} + \cdots\right)$$
$$- \sigma\left(c_0^{'} + hc_1^{'} + h^2 c_2^{'} + \cdots\right)\Bigg] = 0. \tag{72}$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : e_0^{'} = 0, \tag{73}$$
$$h^1 : e_1^{'} + (\mu + \pi)\, e_0 - \sigma c_0 = 0, \tag{74}$$
$$h^2 : e_2^{'} + (\mu + \pi)\, e_1 - \sigma c_1 = 0. \tag{75}$$

Apply HPM 0n equation (37) again and obtained

$$(1-h)\,\frac{dH}{dt} + h\left[\frac{dH}{dt} + \mu H - \kappa S - \theta\varepsilon R\right] = 0. \tag{76}$$

Substitute (40) and (41) into (75)

$$\begin{aligned}
&(1-h)\left(f_0^{'} + hf_1^{'} + h^2 f_2^{'} + \cdots\right) + h\Bigg[\left(f_0^{'} + hf_1^{'} + h^2 f_2^{'} + \cdots\right) \\
&+ \mu\left(f_0 + hf_1 + h^2 f_2 + \cdots\right) - \kappa\left(a_0 + ha_1 + h^2 a_2 + \cdots\right) \\
&- \theta\varepsilon\left(w_0 + hw_1 + h^2 w_2 + \cdots\right)\Bigg] = 0, \qquad (77) \\
&\left(f_0^{'} + hf_1^{'} + h^2 f_2^{'} + \cdots\right) + h\Bigg[\mu\left(f_0 + hf_1 + h^2 f_2 + \cdots\right) \\
&- \kappa\left(a_0 + ha_1 + h^2 a_2 + \cdots\right) - \theta\varepsilon\left(w_0 + hw_1 + h^2 w_2 + \cdots\right)\Bigg] = 0. \qquad (78)
\end{aligned}$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : f_0^{'} = 0, \tag{79}$$
$$h^1 : f_1^{'} + \mu f_{{}_0} - \kappa b_0 - \theta\varepsilon w_0 = 0, \tag{80}$$
$$h^2 : f_2^{'} + \mu f_{{}_1} - \kappa b_1 - \theta\varepsilon w_1 = 0. \tag{81}$$

Thus, applying HPM on the last system equations in (37) and obtained

$$(1-h)\,\frac{dR}{dt} + h\left[\frac{dR}{dt} + (\mu + \varepsilon)\, R - (1-\alpha)\,\delta E - \tau C - \pi P\right] = 0. \tag{82}$$

Substitute (42) and (43) into (81)

$$\begin{aligned}
&(1-h)\left(w_0^{'} + hw_1^{'} + h^2 w_2^{'} + \cdots\right) + h\Bigg[\left(w_0^{'} + hw_1^{'} + h^2 w_2^{'} + \cdots\right) \\
&+ (\mu + \varepsilon)\left(w_0 + hw_1 + h^2 w_2 + \cdots\right) - (1-\alpha)\left(b_0 + hb_1 + h^2 b_2 + \cdots\right) \\
&- \tau\left(c_0 + hc_1 + h^2 c_2 + \cdots\right) - \pi\left(e_0 + he_1 + h^2 e_2 + \cdots\right)\Bigg] = 0, \qquad (83)
\end{aligned}$$

$$\left(w_0^{'} + hw_1^{'} + h^2 w_2^{'} + \cdots\right) + h\Big[(\mu+\varepsilon)\left(w_0 + hw_1 + h^2 w_2 + \cdots\right)$$
$$- (1-\alpha)\left(b_0 + hb_1 + h^2 b_2 + \cdots\right) - \tau\left(c_0 + hc_1 + h^2 c_2 + \cdots\right)$$
$$- \pi\left(e_0 + he_1 + h^2 e_2 + \cdots\right)\Big] = 0, \quad (84)$$

Coefficients of the powers of h are expanded, which gives

$$h^0 : w_0^{'} = 0, \quad (85)$$
$$h^1 : w_1^{'} + (\mu+\varepsilon) w_0 - (1-\alpha) b_0 - \tau c_0 - \pi e_0 = 0, \quad (86)$$
$$h^2 : w_2^{'} + (\mu+\varepsilon) w_1 - (1-\alpha) b_1 - \tau c_1 - \pi e_1 = 0. \quad (87)$$

Substitute (42) and (43) into (85)

$$a_1^{'} = \Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0. \quad (88)$$

Applying the initial condition and integrating both sides of the equation $a_1(0)$ obtained

$$a_1 = (\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0)\, t. \quad (89)$$

Thus

$$a_2^{'} = \begin{bmatrix} (1-\theta)\varepsilon\left((1-\alpha) E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon) R_0\right) t \\ -\rho\beta\left((\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0)\right) t C_0 \\ +S_0(\alpha\delta E_0 - (\mu+\tau+\sigma) C_0) t \\ -(\mu+\kappa)(\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0)\, t \end{bmatrix}$$
$$= \begin{bmatrix} (1-\theta)\varepsilon\left((1-\alpha) E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon) R_0\right) \\ -\rho\beta\left((\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0) C_0\right) \\ +S_0(\alpha\delta E_0 - (\mu+\tau+\sigma) C_0) \\ -(\mu+\kappa)(\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0) \end{bmatrix} t. \quad (90)$$

Subsequently, substitute (38) and (39) into (44) obtained

$$S(t) = a_0 + ha_1 + h^2 a_2 + \cdots \quad (91)$$
$$S(t) = \lim_{h\to 1}\left(a_0 + ha_1 + h^2 a_2 + \cdots\right) \quad (92)$$
$$S(t) = a_0 + a_1 + a_2 + \cdots \quad (93)$$

$$S(t) = S_0 + (\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0)\, t$$
$$+ \begin{bmatrix} (1-\theta)\varepsilon\left((1-\alpha) E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon) R_0\right) \\ -\rho\beta\left((\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0) C_0\right) \\ +S_0\left(\alpha\delta E_0 - (\mu+\tau+\sigma) C_0\right) \\ -(\mu+\kappa)(\Lambda + (1-\theta)\varepsilon R_0 - \rho\beta S_0 C_0 - (\mu+\kappa) S_0) \end{bmatrix} \frac{t^2}{2}. \quad (94)$$

Subsequently, by applying the same method to $E(t)$, $C(t)$, $P(t)$, $H(t)$, and $R(t)$, where h convergent or approaching 1 following the above steps and the results for simulations obtained as

$$E(t) = E_0 + (\rho\beta S_0 C_0 - (\mu+\delta)\,E_0)\,t + \begin{bmatrix} \rho\beta\,(S_0\,(\alpha\delta E_0 - (\mu+\tau+\sigma)\,C_0) + (\Lambda + (1-\theta)\,\varepsilon R_0 - \rho\beta S_0 C_0)) \\ -(\mu+\kappa)\,S_0 C_0 - (\mu+\delta)\,(\rho\beta S_0 C_0 - (\mu+\delta)\,E_0) \end{bmatrix} \frac{t^2}{2}. \tag{95}$$

$$C(t) = C_0 + (\alpha\delta E_0 - (\mu+\tau+\sigma)\,C_0)\,t + [\alpha\delta\,(\rho\beta S_0 C_0 - (\mu+\delta)\,E_0) - (\mu+\tau+\sigma)\,(\alpha\delta E_0 - (\mu+\tau+\sigma)\,C_0)]\,\frac{t^2}{2}, \tag{96}$$

$$P(t) = P_0 + (\sigma C_0 - (\mu+\pi)\,P_0)\,t + [\sigma\,(\alpha\delta E_0 - (\mu+\tau+\sigma)\,C_0) - (\mu+\pi)\,(\sigma C_0 - (\mu+\pi)\,P_0)]\,\frac{t^2}{2}, \tag{97}$$

$$R(t) = R_0 + ((1-\alpha)\,E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon)\,R_0)\,t + \begin{bmatrix} (1-\alpha)\,(\rho\beta S_0 C_0 - (\mu+\delta)\,E_0) + \tau\,(\alpha\delta E_0 - (\mu+\tau+\sigma)\,C_0) \\ +\pi\,(\sigma C_0 - (\mu+\pi)\,P_0) \\ -(\mu+\varepsilon)\,((1-\alpha)\,E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon)\,R_0) \end{bmatrix} \frac{t^2}{2}, \tag{98}$$

$$H(t) = H_0 + (\theta\varepsilon R_0 + \kappa E_0 - \mu H_0)\,t + \begin{bmatrix} \kappa\,(\rho\beta S_0 C_0 - (\mu+\delta)\,E_0) - \mu\,(\theta\varepsilon R_0 + \kappa E_0 - \mu H_0) \\ +\theta\varepsilon\,((1-\alpha)\,E_0 + \tau C_0 + \pi P_0 - (\mu+\varepsilon)\,R_0) \end{bmatrix} \frac{t^2}{2}. \tag{99}$$

2.10 Numerical Simulations

In this section, the results of semi-analytical method called HPM were used, and the simulations was carry out with the help of Maple software for analysis.

Table 2 Variables, Values and Sources

Variable	Value	Source
$S(t)$	200,000	Assumed
$E(t)$	60,000	Assumed
C	50,000	Assumed
P	2330	Assumed
R	500	Assumed
H	250	Assumed

Table 3 Parameters, Value and Sources

Parameter	**Value**	**Source**
Λ	85	[2]
ρ	0.036	Assumed
β	0.0234	Assumed
δ	0.2	[2]
σ	0.007	[2]
ε	0.35	[2]
τ	0.4	Assumed
π	0.006	Assumed
κ	0.03	[13]
μ	0.0160	[13]
α	0.3	[13]
θ	0.1	[13]

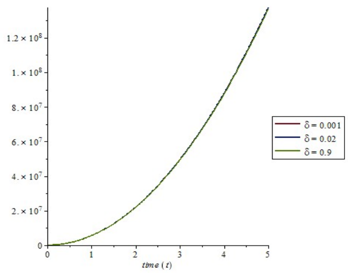

Fig. 2 Graph of corruption showing the solutions of HPM with different rates δ in time.

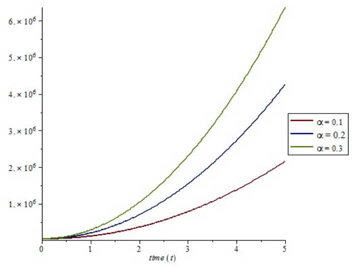

Fig. 3 Graph of exposed individuals that join corruption with different rates α in time.

3 Results and Discussion

This section examines the results of numerical simulations in this paper. In Figure 2, the Corruption graph shows the HPM solutions with different rates δ in time. The graph shows how HPM converges successfully and laps on each other. Also, figure 3 shows the graph of exposed individuals that join Corruption with different rates α in time. The corrupted individuals are exposed to media and public society, which decreases the rate of corrupted individuals in a population by 30 per cent in 5 years. Subsequently, figure 4 shows the graph

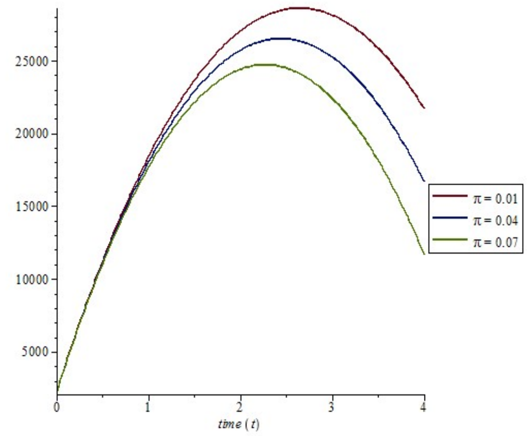

Fig. 4 Graph of punished individuals that join recovered class due to control measure with different rates π in time.

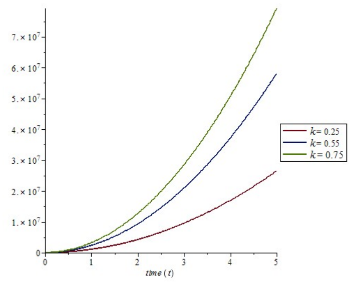

Fig. 5 Graph of susceptible individuals that join honest with different rates κ in time.

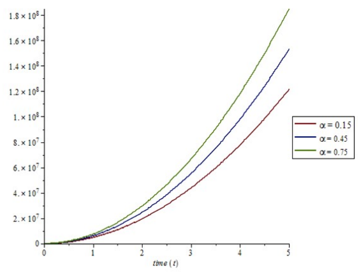

Fig. 6 Graph of recovered individuals with different rates α from exposed class in time.

of punished individuals that join the recovered classes due to control measures with different rates π in time. The graph shows that the more individuals become penalized due to crime or corruption practices, the higher the number of recovered individuals increases and, at the same time, decreases in Corruption.

Figure 5 is a graph of susceptible individuals that join honestly with different rates κ in time. The increase in honest individuals from the graph shows that information and communication technology effectively checks officials holding public and civil organizations' offices, decreasing corruption. Figure 6, the graph of recovered individuals with different rates α from exposed class in time, shows the increment in recovered individuals from corrupt sub populations due to using information and communication technology in every section. Finally, the result shows that when individuals are exposed and punished for the crime committed due to corruption, it drastically decreases corruption. Information and communication technology, media campaigns/awareness, and jail terms for corrupted individuals reduce the practices in society. In future studies, an optimal control strategy will be incorporated.

4 Conclusion

The criminalization of corrupt practices for personal benefit encompasses the public sector, courts, security agencies, the oil and gas sector, and electoral processes. The Homotopy perturbation approach for semi-analytical solutions and the investigation of corruption stability are the main topics of this paper. A deterministic model of Corruption of six (6) compartmentalization was developed and analysed. In summary, the findings demonstrate the accuracy of the Homotopy perturbation approach in handling non-linear terms for semi-analytical results, and they also offer simulations to aid in result interpretation. The level of Corruption was described in the study.

References

1. G. Adamu, M. Bawa, M. Jiya, U. D. Chado, A mathematical model for the dynamics of Zika virus via homotopy perturbation method, Journal of Applied Sciences and Environmental Management, 4, 615–623 (2017).
2. H. Alemneh, Mathematical Modelling Analysis and Optimal Control of Corruption Dynamics, Journal of Applied Mathematics, 2020, 13 (2020).
3. S. N. Aloke, Analyzing Corruption Dynamics and Control Measures in Nigeria: A Mathematical Model, Asian Journal of Pure and Applied Mathematics, 493–511 (2023).
4. Z. K. Birhanu, A. K. Fantaye, Modelling and analysis of corruption dynamics incorporating media coverage. Commun. Math. Biol. Neurosci., 2022, 94 (2022).
5. F. Y. Eguda, A. James, F. A. Oguntolu, D. Onah, Mathematical Analysis of a Model to Investigate the Dynamics of Poverty and Corruption, Abacus, Mathematics Science Series, 44, 352–367 (2019).
6. A. K. Fantaye, Z. K. Birhanu, Mathematical model and analysis of corruption dynamics with optimal control, Journal of Applied Mathematics, 2022, 1–16 (2022).
7. R. I. Gweryina, M. Y. Kura, E. Okwu, An epidemiological model of corruption with immunity clause in Nigeria, World Journal of Modelling and Simulation, 15, 262–275 (2019).
8. J. A. P. Heesterbeck, A brief history of reproduction number and a recipe for its calculations, Acta Biotheoretica, 50, 189–204 (2002).
9. R. S. Isa, S. Emmanuel, N. T. Danat, S. H. Tsok, Mathematical Modelling of Corruption Dynamics: Examining the Reintegration of Formerly Corrupt Individuals, FUDMA Journal of Sciences, 7, 1–13 (2023).

10. H. Ji-Haun, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, International Journal, non-linear Mechanical, 35, 37–43 (2000).
11. M. Jiya, Application of Homotopy Perturbation Method for the Solution of some non-Linear Differential Equations, Pacific Journal of Science and Technology, 11, 268–272 (2010).
12. B. Lang, Framing, localizing anti-corruption norms in transnational civil society organizations: Transparency International in Portugal, Journal of Civil Society, 17, 155–178 (2021).
13. L. Lemecha, Modelling corruption dynamics and its analysis, Ethopian journal of science and sustainable development, 5, 13–27 (2018).
14. N. O. Mokaya, H. T. Alemmeh, C. G. Ngari, G. G. Muthuri, Mathematical modelling and analysis of corruption of morals amongst adolescents with control measures in Kenya, Discrete Dynamics in Nature and Society, 2021, 1–16 (2021).
15. D. R. Merkin, Introduction to the Theory of Stability 24, Springer Science & Business Media, New York (2012).
16. O. M. Nathan, T. A. Haileyesus, G. N. Cyrus, G. M. Grace, Mathematical Modelling and Analysis of Corruption of Morals amongst Adolescents with Control Measures in Kenya, Discrete Dynamics in Nature and Society, 2021, 1–16 (2021).
17. O. M. Nathan, K. O. Jackob, O. M. Nathan, K. O. Jackob, stability analysis stability analysis in a mathematical model of corruption in Kenya, Asian research journal of mathematics, 2019, 1–15 (2019).
18. U. K. Nwajeri, J. K. K. Asamoah, N. R. Ugochukwu, A. Omame and Z. Jin, A mathematical model of corruption dynamics endowed with fractal–fractional derivative, Results in Physics, 52, 106894 (2023).
19. C. Nwosu, Exploring the Effectiveness of Anti-Corruption Laws in Nigeria: A Comparative Study of Legal Frameworks and Practical Outcome, American Journal of Law and Political Science, 2, 67–75 (2023).
20. O. J. Olujobi, Nigeria's upstream petroleum industry anti-corruption legal framework: the necessity for overhauling and enrichment, Journal of money laundering control, 2, 1–27 (2023).
21. N. H. Shah, B. M. Yeolekar, Z. A. Patel, using Incidence Function, Economic Computation & Economic Cybernetics Studies & Research, 2, 165–180 (2017).
22. T. A. Schwetz, T. Calder, E. Rosenthal, S. Kattakuzhy, A. S. Fauci, Opioids and infectious diseases: a converging public health crisis, The Journal of infectious diseases, 220, 346–349 (2019).
23. A. T. Tinuoye, An Overview of Corruption in the Context of the Public Sector, Management Strategies and Tools for Addressing Corruption in Public and Private Organizations, 19–38 (2023).
24. H. Umar, S. Safaria, W. Mudiar, R. B. Purba, H. Harsono, K. Karyaningsih, HU-Model Effectiveness in Corruption Detection, Budapest International Research and Critics Institute-Journal, 4, 10570–10581 (2021).
25. P. Van den Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Mathematical biosciences, 180, 29–48 (2002).